

Formal Education and Public Knowledge

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Abstract

In this paper, I examine the transitional dynamics of a model economy populated by individuals who split their time between acquiring a formal education, producing final goods, and innovating.

The paper has two objectives: (i) to uncover the macroeconomic circumstances that triggered the onset and rise of formal education; (ii) to reconcile the remarkable growth of the education sector with the constancy of other key macroeconomic variables, such as the interest rate, the consumption-output ratio and the growth rate of per capita income (Kaldor facts).

The transitional dynamics of human capital growth models, such as Lucas (1988), would attribute the arrival of education to the diminishing marginal productivity of physical capital. Conversely, the model proposed here suggests that it is the rate of learning that catches up with the rate of return on physical capital. The learning rate increases with the stock of public knowledge – the primary input used by the education sector. The conjecture is that when public knowledge hits a critical threshold, the rate of return on education becomes large enough to induce individuals to spend time in school. The stock of public knowledge grows as the number of technologies available increases. In accordance with the development trajectories of modern economies, the model generates a development sequence in which an innovation-only economy is followed by an innovation-education economy.

The model's transitional paths are matched with about three centuries of U.S. economic data.

Keywords: Public Knowledge, Learning Rate, Transitional Dynamics, Calibration.

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1 Introduction

The average years of schooling of the labor force in the U.S. grew in the period 1840-2000 from 1.14 to 13. A similar remarkable expansion of education is observed in all contemporary high income countries and in most developing countries (Baier et al., 2007; Schofer and Meyer, 2005).

The transitional dynamics of Lucas (1988) imply that the greater allocation of time to education is the consequence either of a decline in the marginal productivity of physical capital or of an improvement in the ability to learn. In the early stages of development investments in physical capital yield a higher return than investment in human capital, and there is therefore no incentive for individuals to go to school. Later on, as physical capital expands its rate of return declines, and resources are diverted into human capital accumulation. The data do not show, however, any significant long-term decline in capital returns. The calculation of Barro (2006) and Siegel (1998) suggest that the average real bill and stock returns in the 19th century are about the same as those recorded in the 20th century.¹ Furthermore, there is no evidence of changes in human neurobiology to account for an enhanced ability to absorb knowledge.

A second branch of the literature emphasizes the historical role played by the industrial revolution both on the demand and on the supply of skills. The accumulation of capital and the diffusion of new techniques during the process of industrialization made skills more valuable (demand); at the same time income rose above the subsistence level, which allowed families to invest more in human capital (supply).² Hence, the reallocation of time toward human capital formation could be associated with a rise in income rather than with a decline in the marginal productivity of physical capital. The problem with this argument, however, is that as wages go up, the actual and the opportunity cost of education increase as well, and, indeed, there are historical episodes suggesting that technological progress did not directly lead to more formal education. In fact, the industrial revolution brought about a considerable rise in wages for the British workers, who remained virtually illiterate until the second half of the 19th century. Easterlin's (1981) data show that a noticeable increase of British primary education occurred only in the second half of the 19th century, almost a century after the onset of the Industrial Revolution (henceforth IR).³

¹Table IV of Barro (2006) shows that the real stock return in the United States for the 1880-2004 and 1954-2004 time periods are 0.081 and 0.089 respectively. The real bill return for the same two periods is 0.015 and 0.017, respectively. Siegel (1998) calculates a return of 7% for the periods 1802-1997, 1871-1997 and of 6.7% for the period 1913-1997 (see Table 8-1).

²See Galor (2005) for a comprehensive discussion.

³In the first phase of the IR, literacy was generally higher in the rural areas than in industrial towns (Kirby 2003, p.116), and educationists found it difficult convince parents employed in industry that their children would benefit from schooling (Stephens, 1998, p. 19). Of course, some human capital formation was

What, then, was the mechanism that tilted the balance of investment to tilt unequivocally in favor of education? I will argue that a critical factor in the onset of modern education was the expansion of public knowledge. The hypothesis is that the creation and dissemination of new technologies raises the stock of public knowledge. When this reaches a critical threshold, the rate of learning becomes large enough relative to the return on physical capital; consequently, individuals then find it optimal to invest time in education.

The main conjecture rests on three observations. First, public knowledge is the main input of the education sector. By public knowledge I mean the content of information goods, such as books, electronic files, drawings, and artifacts, that can be studied for the sake of solving production problem more efficiently or for generating new ideas. Second, technological advances unfold new public knowledge. When a new kind of bridge is built, the frontier technical knowledge is pushed forward, for it becomes feasible to connect geographical areas that were once isolated. Other individuals can replicate the bridge by studying its blueprint, or by learning directly the technique from the constructors of the original bridge. Likewise, when a firm adopts a new principle of organization, interested individuals have more information about the ways production can be carried out. Third, innovation in the education system is driven by technological advances. When an innovation is deemed to be important enough, existing textbooks on the subject are updated, new textbooks are written, and sometimes entire new schools are established. Indeed, a great deal of educational innovation occurred in the past century, and, arguably, most of it is associated with technological progress. For instance, advances in the industries of electricity, broadcasting and communication, as well as the spread of radars, guided missiles and control systems, prompted a number of reforms in the electrical engineering curricula (Terman,1998 [1976]).⁴

The interaction between technological progress, education, and public knowledge will be analyzed within a growth model in which the representative individual decides how to optimally allocate time between production, schooling, and innovation activities. One novelty of the model is the incorporation of a mechanism whereby innovation activities generate positive externalities that benefit the education sector: as consequences of such activities, current students can tap into a set of public knowledge that is larger than the one available to previous generations of students.⁵The focus of the analysis is on the transitional dynamics

going on in the UK during the industrial revolution. The British strategy, however, was to learn by doing. This had worked well enough so long as technology remained an accretion of improvements and invention based on known techniques (Landes, 1998, Ch. 18).

⁴Similarly, computer science departments have boomed since the arrival of information technologies. In medicine, clinical simulations became part of medical doctor training after the introduction of new medical instruments, such as part-task simulators, cardiovascular systems, and multimedia programs. (Bradley, 2006). A further noticeable innovation in medical education is associated with the dissemination of videoscopic imaging techniques (Borst, 2001).

⁵Diamond (1997) and Aiyar et al. (2008) points out that, in pre-industrial societies, there have been

of the model economy. In a pre-modern development stage, the expected reward of entrepreneurial activities is large enough to induce people to adopt new technologies, but the potential rate of learning is still poor and does not compensate for the schooling opportunity costs (missed wages or entrepreneurial profits). As technology advances, new knowledge unfolds and, as a result, the learning rate goes up. This positive externality enjoyed by the education sector is the key element that explains why returns on education catch up with returns on physical capital. One important feature of the model's transitional dynamics is that the interest rate plays a negligible role in driving the economy towards the balanced growth path, for the accumulation of public knowledge prevents the decline of the marginal productivity of capital that would otherwise be observed. Furthermore, such a decline in the interest rate is not needed to induce individuals to invest in school, because the education function shifts up continuously driven by the greater availability of public knowledge.

The conjecture provided in this paper for the expansion of education complements theories that, following Ben-Porath's (1967) insight, see human capital formation in connection with the rise in life expectancy (Boucekkine, de la Croix and Licandro (2002, 2003), Cervellati and Sunde (2005), Soares (2005)). Indeed, the time series of longevity and education have been moving in lock-step since the middle of the nineteenth century. However, the recent work by Acemoglu and Johnson (2006), which finds no effect of life expectancy on schooling, suggests that the serial correlation between education and longevity may be spurious. Furthermore, Hazan and Zoabi (2006) argue that, in principle, parents' choices about their children's levels of education may be not be affected by longevity, for this raises not only the return on education but also that on fertility. Hence, the parents may be tempted to increase the future stream of wages of the household by having more children rather than investing more on their children's education.

This paper generalizes existing growth models in which income expansion is driven both by investment in innovation and investment in education. Arnold (1998), and Funke and Strulik (2000), and Lloyd-Ellis and Roberts (2002) propose models that merge the view that the growth of modern economies is based on the accumulation of human capital (Uzawa (1965), Lucas (1988), and Rebelo, (1991)) with the view that emphasizes R&D investments (Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992)). Here, I go one step further and allow the education sector to benefit directly from the knowledge generated by the innovation sector.⁶ Kosempel (2004) also links long-run stylized facts with

several historical instances in which technology regressed. However, temporary setbacks have not stopped the process of knowledge creation. Although some of the Greek and Roman architecture was no longer in use during the Middle Age, the knowledge remained embodied in artifacts, and the Renaissance architects—most notably Brunelleschi—could study them.

⁶In Lloyd-Ellis and Roberts (2002), disembodied knowledge is assumed to be proportional to the stock of human capital of an earlier generation, whereas the behavior of human capital and public knowledge emerges

features of the transitional dynamics of a growth model with two engines of growth. The main difference is the way resources are allocated to the innovation sector. In Kosempel (2004), firms allocate a constant fraction of output to research and development, whereas in my economy this fraction is included in the household’s list of choice variables. Enlarging the set of choices in this direction allows me to highlight a substitution of innovation for education time. My analysis bears a resemblance to Acemoglu and Guerrieri (2008), and Kongsamut, Rebelo, and Xie (2001) in that I try to account for a major structural change – the diversion of resources to the school sector – in an economy characterized by the constancy of key macroeconomic variables such as the output-capital ratio, share of labor income, the interest rate, and the growth rate of output – known as Kaldor facts. Here however the Kaldor facts are reproduced as features of the transitional dynamics of an economy that tends towards a balanced growth path rather than as those of a nonbalanced growth economy. This paper also relates to one insight of Nelson and Phelps (1966): the return to education is greater in technological dynamic economies.⁷ It departs from the Nelson and Phelps framework because the return on education is not a function of the gap between the technological frontier and the technology used in production – it is rather a function of the menu of technologies. Their approach is useful when studying contemporary economies at different stages of development, but it is more problematic in an historical perspective, for it is difficult to assess whether such a gap was smaller at the time of Leonardo – implying low returns on education – than it is today.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 characterizes the dynamics of the macroeconomic competitive equilibrium of this economy, and shows how the balanced growth path (henceforth BGP) is sensitive to variations in key parameters. Section 4 linearizes the system around the steady state and studies how the economy on its BGP reacts to a variety of shocks. Section 5 undertakes a calibration of the model economy to investigate whether the dynamics generated by the model are consistent with the historical rise of the education sector and with other key macroeconomic long-run U.S. time series. Section 6 discusses how the main features of the dynamics relate to historical facts, and extends the calibration back in time to include the period prior to the onset of formal education. Section 7 shows how the model is sensitive to variations in two key parameters. Section 8 concludes. Appendix A shows the conditions for the existence of a BGP. Finally, Appendix B solves the model under parameter restrictions and obtains existing models as special cases.

from the model’s dynamics in my model.

⁷The literature focused more on another point of the paper, namely that education helps reduce the gap between the technological frontier and the actual one. Benhabib and Spiegel (2005) summarizes empirical attempts to test this hypothesis.

Table 1: Description of Parameters and Variables

<i>Parameters</i>	
z	productivity index, final goods sector
α_1	output elasticity to homogenous capital
α_2	output elasticity to intermediate goods
γ	elasticity of substitution between intermediates
θ	productivity index, innovation sector
b	productivity index, education sector
β ($\tilde{\beta}$)	externality of public knowledge (human capital), innovation sector
ϕ ($\tilde{\phi}$)	externality of public knowledge (human capital), education sector
σ	inverse of the intertemporal elasticity of substitution in consumption
ρ	subjective discount rate
<i>Variables</i>	
$u_P/u_E/u_I$	production/education/innovation time
k_1	homogenous capital
k_2	aggregate index of intermediate goods
y	final good
$x(j)$	quantity of intermediate good j
h	human capital
n	number of intermediate goods/public knowledge
$\psi \equiv \frac{h}{n}$	ratio of human capital over public knowledge
$\chi \equiv c/k_1$	consumption-capital ratio
p_{k_2}	price of k_2
r	real interest rate
w	wage rate (per unit of human capital)
π	flow of profit of intermediate firm
v	market value of intermediate firm
λ	shadow value of income
μ	shadow value of human capital
g_x	x 's growth rate
\dot{x}	x 's time derivative

2 The Model

The economy is populated by infinitely-lived individuals of size 1. There is no population growth. Each individual is endowed with one unit of time that he allocates between three types of activity: final good production (u_P), education (u_E), and innovation (u_I). Hence, the following constraint holds:

$$1 = u_P + u_E + u_I. \quad (1)$$

Final Good Production. The flow of final good is given by

$$y = zk_1^{\alpha_1} k_2^{\alpha_2} (hu_p)^{1-\alpha_1-\alpha_2}, \quad (2)$$

where z is a positive constant, h denotes the level of skills, k_1 is the service of physical capital, and k_2 is an aggregate measure of intermediate inputs, namely $k_2 = [\int_0^n x(j)^\gamma dj]^{1/\gamma}$, where $x(j)$ denotes the quantity of intermediate good j , and γ regulates the elasticity of substitution between intermediates. There are n such intermediates that can be used for production. The elasticity of output with respect to the two types of physical capital are given by α_1 and α_2 . Let r be the rental price of k_1 , p_{k_2} be the price of k_2 , and w be the wage rate of one unit of human capital. The demand schedule for the three inputs is:

$$r = \alpha_1 y / k_1, \quad (3)$$

$$p_{k_2} = \alpha_2 y / k_2,$$

and

$$w = (1 - \alpha_1 - \alpha_2) y / (hu_p), \quad (4)$$

respectively. The price of the final good is normalized to one.

Intermediate Goods. One unit of intermediate input is obtained by means of one unit of final output. There are no fixed costs. Contrary to physical capital, intermediate goods are embodied in the final output, implying that the marginal cost is one and that the price of $x(j)$ is $1/\gamma$ for any $j \in [0, n]$ – monopolistic competition price. From the symmetry across intermediates it follows that $x(j) = x$ for any $j \in [0, n]$, that $p_{k_2} k_2 = nx/\gamma$, and that $k_2 = n^{1/\gamma} x$. Therefore, the demand for an intermediate input and the intermediate producer's profit, π , can be expressed as a function of final output, namely $x = \alpha_2 \gamma y / n$, $\pi = (1 - \gamma) \alpha_2 y / n$ and Eq. (2) can be reduced to

$$y = \tilde{z} k_1^{\alpha_1/(1-\alpha_2)} n^{(1/\gamma-1)\alpha_2/(1-\alpha_2)} (hu_p)^{(1-\alpha_1-\alpha_2)/(1-\alpha_2)}, \quad (5)$$

where $\tilde{z} = z^{1/(1-\alpha_2)} (\alpha_2 \gamma)^{\alpha_2}$.

Education. The formation of human capital is given by

$$\dot{h} = bu_E h^{\tilde{\phi}} n^{\phi}, \quad (6)$$

where $b > 0$ is a learning parameter, u_E is learning time, n is an index that captures the stock of public knowledge, assumed to be proportional to the menu of technologies, and h is human capital. The parameters ϕ and $\tilde{\phi}$ are the elasticities of the flow of human capital to the stock of public knowledge and to the stock of human capital respectively. Both parameters are smaller than one. Typically the skills acquired through schooling are a function of the time spent in school and a positive externality from investment made in knowledge by previous generations. This is for instance the case in Uzawa (1965), Lucas (1988), Stokey (1991), Bils and Klenow (2000), Becker et al. (1990). To emphasize that knowledge acquisition is a social learning process Tamura (1991) introduces an additional externality in the individual's learning function that is represented by the average human capital of the population. Here I focus on the greater opportunities associated with the expansion of the frontier knowledge rather than with the knowledge of the typical individual. Of course, if the set of public knowledge and the average level of human capital expand in the same proportion the alteration of the education function proposed here would not add much to the analysis. Indeed this is a reasonable simplification when analysis is conducted on a balanced growth path (see Lloyd-Ellis and Roberts (2002)). Public knowledge and average human capital do not, however, need to go hand in hand. The scientific revolution of the seventeenth century and the advances in chemistry, biology, medicine and other areas that occurred during the Industrial Revolution have led to a great expansion of the set of public knowledge, but these advances did not translate into a major increase in the level of skill level of the typical worker, which was probably not substantially different from that of a worker in the Middle Ages.

Innovation. An individual with human capital h that spends u_I of his or her time working as an entrepreneur generates a flow of innovation

$$\dot{n} = \theta u_I h^{\tilde{\beta}} n^{\beta}, \quad (7)$$

where $\theta > 0$, whereas β and $\tilde{\beta}$ are non-negative and smaller than one. The above specification allows to obtain some of the existing models (for a review see Jones, 1999, 2005, and Klenow and Rodriguez-Clare, 2005) as special cases. In particular, the functional form is very close to non-scale growth model that follow Jones (1995) (see also Arnold, 2006), except that here the existence of two engines of growth requires the additional constant-return-to-scale restriction for a balanced growth path to exist. In Romer (1990) $\beta = 1$ and h is constant (a change in human capital would be captured by a variation in θ). Conversely, in Grossman and Helpman (1991, Ch. 3.1) $\beta = 0$ – there are no dynamic R&D spillovers.

Households. Let c denote consumption and ρ the subjective discount rate. The aim of the individual is to find a set of control functions $(c(t), u_P(t), u_E(t), u_I(t))$, where t denotes time, that maximizes the utility function $\int_0^{+\infty} u(c(t)) \exp(-\rho t) dt$, provided that (1), (6) and the dynamic asset budget constraint, $\dot{a}(t) = w(t)u_P(t)h(t) + r(t)a(t) + \theta u_I(t)n(t)^\beta h(t)^{\tilde{\beta}} v(t) - c(t)$, are satisfied for a given initial condition on assets and human capital (there are no profits distributed to households). The variable $v(t)$ represents the value of a capital good firm, $a(t)$ indicates the per capita amount of assets, and $r(t)$ denotes the real interest rate. The first term on the right of the equality is labor income, the following term captures interest income, and the third one accounts for the entrepreneurial gain of establishing new intermediate good firms. I formulate the optimization problem as a Hamiltonian system that includes the dual variables λ and μ , which are the shadow values associated with the asset budget constraint and (6) respectively. Assuming $u(c) = (c^{1-\sigma} - 1)/(1 - \sigma)$, where $\sigma > 0$ is a parameter that captures the inverse of the intertemporal elasticity of substitution in consumption, the current value Hamiltonian for the household is

$$H(a, h; \lambda, \mu; c, u_E, u_I) = \frac{c^{1-\sigma} - 1}{(1 - \sigma)} e^{-\rho t} + \lambda[w(1 - u_E - u_I)h + \theta u_I n^\beta h^{\tilde{\beta}} v + ra - c] + \mu b u h^{\tilde{\phi}} n^\phi.$$

where I have used (1) to replace u_P and dropped the time variable (t). The shadow values λ and μ evaluate increments of income and of human capital in units of today's utility, respectively. The objective is to find a four-dimensional vector (λ, μ, a, h) that maximizes $H(\cdot)$. Below, I report the first-order necessary conditions for an interior solution and the conditions prevailing in a corner solution when u_I or u_E are equal to zero.

The condition with respect to c is

$$c^{-\sigma} e^{-\rho t} = \lambda, \tag{8}$$

and the one with respect to u_I is

$$wh = \theta n^\beta h^{\tilde{\beta}} v \text{ and } u_I > 0, \tag{9a}$$

or

$$wh > \theta n^\beta h^{\tilde{\beta}} v \text{ and } u_I = 0. \tag{9b}$$

Likewise, the optimal condition on u_E is

$$\lambda wh = \mu b n^\phi h^{\tilde{\phi}} \text{ and } u_E > 0, \tag{10a}$$

or

$$\lambda wh > \mu b n^\phi h^{\tilde{\phi}} \text{ and } u_E = 0, \tag{10b}$$

Table 2: The Four-Dimensional System

$g_\psi = bu_E(\psi)^{-\phi} - \theta u_I(\psi)^{1-\beta}$	(13)
$g_\chi = \left(\frac{1}{\sigma} - \frac{1-\alpha_2\gamma}{\alpha_1}\right)r + \chi - \frac{\rho}{\sigma}$	(15)
$g_r = -\frac{1-\alpha_1-\alpha_2}{\alpha_1}g_w + (1/\gamma - 1)\frac{\alpha_2}{\alpha_1}g_n$	(17)
$g_{u_P} = -\left(\frac{1-\alpha_2}{\alpha_1}\right)g_w + (1/\gamma - 1)\frac{\alpha_2}{\alpha_1}g_n + \frac{1-\gamma\alpha_2}{\alpha_1}r - \chi - g_h$	(19)

Note. The table contains the equations that describe the four-dimensional system in r, ψ, χ, u_P , provided that g_n, g_h , and g_w are replaced with expressions derived from (7), (6), and (18), that is, $g_n = \theta u_I \psi^{1-\beta}$, $g_h = bu_E \psi^{-\phi}$, $g_w = r - \frac{1-\gamma}{1-\alpha_1-\alpha_2} \alpha_2 \theta (\psi)^{1-\beta} u_P - \beta [bu_E(\psi)^{-\phi} - \theta u_I(\psi)^{1-\beta}]$ and that u_I and u_E are eliminated through (1) and (22).

If $u_E > 0$, the following condition must also hold on the optimum trajectory

$$-\dot{\mu} = \lambda w(1 - u_E - u_I) + \tilde{\beta} \lambda \theta u_I n^\beta h^{\tilde{\beta}-1} v + \tilde{\phi} \mu b u(h)^{\tilde{\phi}-1} n^\phi. \quad (11)$$

Finally, the condition on assets is

$$-\dot{\lambda} = \lambda r \quad (12)$$

Hence Eqs. (8), (9a), (10a), (11), (12), along with two transversality conditions ($\lim_{t \rightarrow +\infty} \lambda(t)a(t) = \lim_{t \rightarrow +\infty} \mu(t)h(t) = 0$) represent the necessary conditions of the household's dynamic problem with initial endowment (a_0, h_0) . A sufficient condition for a solution of the first-order conditions to solve the optimization problem is that the Hamiltonian $H(\cdot)$, evaluated when the conditions for the control variables (8), (9a) and (10a) hold, be jointly concave in (a, h) .⁸ Inserting these three equations into $H(\cdot)$, we obtain

$$H^* = \kappa + \tilde{\lambda} v \theta n^\beta h^{\tilde{\beta}} v + \tilde{\lambda} r a$$

where $\kappa = \frac{\tilde{\lambda}^{\sigma/(1-\sigma)} - 1}{(1-\sigma)} e^{-\rho t} - \tilde{\lambda}^{\sigma/(1-\sigma)}$ and $\tilde{\lambda} = \lambda e^{-\rho t}$. Notice that H^* is clearly concave in a and h .

3 Reduced Dynamic System

This section describes the macroeconomic competitive equilibrium when both the innovation sector and the education sector are already in place ($u_I > 0$ and $u_E > 0$). The competitive equilibrium is obtained by combining the optimum conditions from the production side – (3) and (4) and (5) – with the household's optimum conditions described above, subject to the resource constraint $\dot{k}_1 = y - c - nx (= y(1 - \alpha_2\gamma) - c)$. In order to obtain a system that converges to a Balanced Growth Path (BGP), an equilibrium in which n, c, k_1, h grow

⁸This is known as the Arrow theorem. See Kamien and Schwartz, p. 222.

at a constant rate and in which r, u_I, u_E are constant, some restrictions are needed on the parameters $\beta, \tilde{\beta}, \phi$, and $\tilde{\phi}$. In particular, Eq. (6) and Eq. (7) imply that on the BGP

$$\phi g_n = (1 - \tilde{\phi}) g_h$$

and that

$$(1 - \beta) g_n = \tilde{\beta} g_h.$$

Dividing the two equations side by side we get the constraint

$$\frac{\phi}{(1 - \tilde{\phi})} = \frac{(1 - \beta)}{\tilde{\beta}}.$$

For the sake of simplicity, I will assume that $\tilde{\phi} = 1 - \phi$ and $\tilde{\beta} = 1 - \beta$. Let $\chi \equiv c/k_1$ and $\psi \equiv \frac{h}{n}$. With this transformation, the competitive equilibrium can be expressed as a four-dimensional system in r, ψ, χ , and u_P . The dynamics of special cases in which β or ϕ is zero or one are considerably simpler. Appendix B briefly illustrates three of such cases and relates them to the extant literature. In the following analysis, however, the two parameters can assume any value within the unit interval. In what follows, four differential equation are obtained.

The behavior of the ratio ψ is given by Eqs. (6) and (7):

$$g_\psi = bu_E(\psi)^{-\phi} - \theta u_I(\psi)^{1-\beta}. \quad (13)$$

The resource constraint can be written as

$$g_{k_1} = \frac{(1 - \alpha_2 \gamma)}{\alpha_1} r - \chi. \quad (14)$$

From the household problem one obtains $g_c = \frac{1}{\sigma}(r - \rho)$. This, combined with (14), yields

$$g_\chi = \left(\frac{1}{\sigma} - \frac{1 - \alpha_2 \gamma}{\alpha_1} \right) r + \chi - \frac{\rho}{\sigma}. \quad (15)$$

The reduced-form production function (5) implies that

$$g_y = \frac{1 - \alpha_1 - \alpha_2}{1 - \alpha_2} (g_h + g_{u_P}) + \frac{(1/\gamma - 1)\alpha_2}{1 - \alpha_2} g_n + \frac{\alpha_1}{1 - \alpha_2} g_{k_1}, \quad (16)$$

which, combined with the time-log differentiated versions of Eqs. (4) and (3), and with Eq. (14), delivers

$$g_r = -\frac{1 - \alpha_1 - \alpha_2}{\alpha_1} g_w + (1/\gamma - 1) \frac{\alpha_2}{\alpha_1} g_n. \quad (17)$$

where the growth rate of n is given in Eq. (7) and that of w is derivable either from Eq. (18) or from Eq. (21) reported below. The labor market equilibrium condition (9a)

implies that $g_w + \beta g_h = \frac{\dot{v}}{v} + \beta g_n$. Since $\frac{\dot{v}}{v} = r - \frac{\pi}{v}$ and $wh^\beta = \theta n^\beta v$, it follows that $\frac{\dot{v}}{v} = r - \frac{1-\gamma}{1-\alpha_1-\alpha_2} \alpha_2 \theta \psi^{1-\beta} u_P$. Consequently,

$$r - g_w = \frac{1-\gamma}{1-\alpha_1-\alpha_2} \alpha_2 \theta (\psi)^{1-\beta} u_P + \beta g_\psi. \quad (18)$$

From Eqs. (14), (18), and (16), one gets

$$g_{u_P} = -\left(\frac{1-\alpha_2}{\alpha_1}\right)g_w + (1/\gamma - 1)\frac{\alpha_2}{\alpha_1}g_n + \frac{1-\gamma\alpha_2}{\alpha_1}r - \chi - g_h. \quad (19)$$

Notice that (13), (15), (17), and (19) is already a four-dimensional system in r, ψ, χ , and u_P , provided that g_n, g_h , and g_w are replaced with expressions derived from (7), (6), and (18). However, the system also contains u_E and u_I and therefore two additional relationships are needed. One is given by the time constraint (1). The other is obtained by exploiting the link between the two shadow values λ and μ . By inserting Eq. (10a) into Eq. (11) and noting that $v = wh^\beta/\theta n^\beta$ we get

$$-\dot{\mu}/\mu_t = b(\psi)^{-\phi}(1 - \phi u_E - \beta u_I). \quad (20)$$

This, combined with the log-differentiated version of the Eq. (10a), yields

$$r - g_w = b(\psi)^{-\phi}(1 - \phi u_E - \beta u_I) + \phi(g_h - g_n). \quad (21)$$

Because the left-hand side of this equation and that of Eq. (18) are the same, it follows that

$$(\beta - \phi)(g_h - g_n) = b(\psi)^{-\phi}(1 - \phi u_E - \beta u_I) - \frac{1-\gamma}{1-\alpha_1-\alpha_2} \alpha_2 \theta (\psi)^{1-\beta} u_P. \quad (22)$$

For a given u_I and ψ , this relationship pins down the value of u_E .

The competitive equilibrium is in fact represented by a fourth-order dynamic system over the space (r, ψ, χ, u_P) and the two relationships (1) and (22).⁹ Table (2) summarizes the system.

3.1 Balanced Growth Path and Comparative Dynamics

The balanced growth path is defined as an equilibrium in which consumption, output, physical capital, human capital and public knowledge grow at a steady rate (but not necessarily at the same rate) and in which the interest rate and the fraction of time allocated to education and innovation are constant. By setting the left-hand-side of Eqs. (15), (17), (19), (13),

⁹One would also need to verify that the transversality conditions hold. Alternatively, one could compute the steady state and make sure that the equilibrium trajectory tends to that point (see Kamien and Schwarts, 1991, p.174). Here, it is easy to follow the latter route.

and (22) to zero and eliminating g_w through Eq. (18) one obtains a system whose solution represents the steady state of six stationary variables. The appendix derives the conditions for such an equilibrium to exist.

Because it is hard to carry out comparative dynamics analytically on this system, I study how the steady state reacts to variations in technology and preferences through a number of simulations. The main results of the experiments are summarized in Table (4). The set of baseline parameters, reported in Table (3), implies an interest rate of 5%, an annual rate of growth of output of 1.5%, and a labor income share – calculated from Eq. (5) – of 0.7.

Preferences. An economy with a high discount rate (ρ) has a relatively low saving rate. Indeed, the second column of Table (3) shows a positive sign associated with χ – which is inversely related to the interest rate. In a broad sense, u_E and u_I are also part of the saving decision. The relatively low level of broad saving implies slower output growth and higher interest rate (because of the relatively low level of physical capital). An increase in σ – the inverse of the intertemporal elasticity of substitution – yields similar qualitative results.

Innovators' productivity (θ). A rise in this parameter clearly has a positive effect on u_I . As a result, the economy accumulates more public knowledge. Since this is a costless input for the education sector, individuals will also spend more time in school. Nevertheless the sign associated to ψ is negative, indicating that human capital does not increase as much as public knowledge. Because both innovation and education time increase, output growth unequivocally goes up. Both the interest rate and χ also increases as resources are shifted away from physical capital accumulation.

Quality of Education (b). An exogenous enhancement of the quality of education induces people to spend longer stretches of time in school. As a result, individuals stock up more human capital. This leads to greater productivity in the final goods sector as long the displacement effect on physical capital is limited, and it eventually shifts the demand for intermediate goods upward. The prospective of higher future profits causes an appreciation in the value of intermediate firms. The higher return on innovation activities leads to a higher u_I . The economy grows faster because both u_I and u_E rise. As in the previous experiment, innovation and education displace investments in physical capital. Hence the signs associated with χ and r are positive.

The elasticity ϕ . If the elasticity of education to public knowledge increases, then the elasticity to human capital declines. Consequently, human capital loses value, for it plays a more modest role in promoting human capital formation in the future. The lower skill level of the work force diminishes the demands for new products, which leads to a reduction in u_I . The negative sign associated with ψ indicates that the decline of human capital is relatively greater than that of public knowledge. Clearly, the economy grows at a slower pace. Finally, the interest rate and χ also decline because more resources are diverted into physical capital

investments.

The elasticity β . The same comment I made for ϕ also applies to β with the only caveat that now ψ moves in the opposite direction.

Output elasticity to capital (α_1). Since output is produced with a constant return to scale technology, when α_1 rises the output elasticity to labor, adjusted for skills, drops. The outcome of such a variation is ambiguous. On the one hand, human capital is less valuable in production. But this negative effect is compensated for by the greater productivity of physical capital. When departing from low (high) values of α_1 the physical-capital effect (human-capital) dominates and u_E rises (drops). u_I is only indirectly affected by the shock (through the market variation of firms values), and its behavior is qualitatively similar to that of u_E . Physical capital becomes relatively more important; therefore, the signs associated with r and χ are negative.

Output elasticity to intermediate goods (α_2). Some of the consequences of a rise in α_2 associated with the labor share are similar to those just illustrated for α_1 . Here, however, the quantitative impact on u_I is greater because the indirect effect generated by human capital comes on the top of a direct one. Furthermore, when α_2 is high, homogeneous capital loses ground relative to intermediated goods. This explain the positive relationship linking α_2 both to r and χ .

The elasticity across intermediate goods (γ). When the parameter γ increases, the monopoly power of intermediate firms diminishes. Clearly u_I drops, but the effect on education is ambiguous because education partly replaces innovation. Hence, if the cutback on innovation substantially reduces the development of public knowledge, the return of education declines and so does education time. This explains the inverted-U relationship between schooling time and γ .

4 Linearization around the steady state

In order to gain further insights into the dynamics of the model, I will study the economy's adjustment process around the steady state. I will focus on the dynamics of four key variables r, ψ, u_P , and χ because once their behaviors are known, the patterns of u_E and u_I can be easily obtained through (1) and (22). The fourth-order dynamic system is given by Eqs. (15), (17),(19), and(13), provided that the expressions g_h, g_n , and g_w are replaced according to Eqs. (6), (7), and (18). The system consists of two jumpy variables, u_P and χ , and two predetermined variables; namely, the human capital-knowledge ratio, ψ , and the interest rate r . The interest rate is proportional to the output-capital ratio (see Eq.(3)) which, in turn, is a function of the choice variable u_P . Since u_P is already part of the list of jumpy

variables, r will be considered a state-like variable.¹⁰ The distinction between jumpy and predetermined variables is relevant for establishing the local saddle-stability properties of the systems around the steady state. Because there are two predetermined state variables, r and ψ , with initial value $r(0)$, and $\psi(0)$, this type of stability requires that the linearized system in the neighborhood of the steady state be a saddle with two-dimensional stable and two-dimensional unstable manifolds. A few simulations reveal the following patterns. If $0 < \phi < \beta < 1$, the Jacobian may have two positive and two negative eigenvalues, all real (case 1.a), or two real and positive eigenvalues and two complex conjugate eigenvalues with negative real parts (case 1.b). If $0 < \beta < \phi < 1$ we have two situations: one eigenvalue is negative and real and three are real and positive (2.a); or one eigenvalue is negative and real, a second one is positive and real, and the other two are complex and conjugates (case 2.b).

The Stable Manifold Theorem guarantees, in cases (1.a) and (1.b), the existence of a two-dimensional stable manifold and a two-dimensional unstable manifold (Palis and DeMelo, 1982). In situation (1.b), the system generates oscillating dynamics that are difficult to match with data. In cases (2.a) and (2.b), the stable manifold has only one dimension. Hence the system is unstable because it has two predetermined variables. Therefore, I will continue the exposition assuming that the parameters are in a set compatible with case (1.a). Notice that the system does not give rise to indeterminacy, a situation with three negative eigenvalues. Relaxing the assumption of constant return to scales in one or more of the three sectors, however, might give rise to indeterminacy (Benhabib and Perli, 1994).¹¹

Let ω_1 and ω_2 be the two stable eigenvalues, with $\omega_2 < \omega_1 < 0$, and let \tilde{v}_1 and \tilde{v}_2 be the four-dimensional vector associated with the two negative eigenvalues ω_1 and ω_2 , respectively. The ordering of the variables from the top down is: r, ψ, u_P, χ . Then, the generic form of the stable solution for r and ψ is given by

$$r(t) - r^* = B_1 \tilde{v}_{11} e^{\omega_1 t} + B_2 \tilde{v}_{12} e^{\omega_2 t}, \quad (23)$$

and

$$\psi(t) - \psi^* = B_1 \tilde{v}_{21} e^{\omega_1 t} + B_2 \tilde{v}_{22} e^{\omega_2 t}, \quad (24)$$

where \tilde{v}_{ji} denotes the j -th element of vector i and B_1 and B_2 are constants dependent on the distance of the initial position from the steady state and on the eigenvalues (they are calculated from the previous two equations by setting $t = 0$).

¹⁰One could build a dynamic system in which the state variable is the interest rate net of u_p , but the graphical illustrations would be less intuitive.

¹¹The advantage of indeterminacy in this context would be that the model might explain why two countries with similar initial condition on the interest rate and ψ , choose distinct patterns of consumption, education, and innovation.

To learn how the economy approaches the steady state it is useful to divide (23) and (24) side by side, which gives us the slope of the transitional path in the space $(r - \psi)$, that is

$$\frac{\Delta\psi(t)}{\Delta r(t)} = \frac{B_1\tilde{v}_{21}e^{\omega_1 t} + B_2\tilde{v}_{22}e^{\omega_2 t}}{B_1\tilde{v}_{11}e^{\omega_1 t} + B_2\tilde{v}_{12}e^{\omega_2 t}},$$

where $\Delta r(t) \equiv r^* - r(t)$ and $\Delta\psi(t) \equiv \psi^* - \psi(t)$. Although the slope is time-varying and depends on the starting point (through B_1 and B_2), it converges to v_{21}/v_{11} as $t \rightarrow +\infty$, regardless of the starting point. Thus, all trajectories approach the steady state along the same direction.

Fig. (1) shows that every trajectory starting from any point in the positive quadrant $(r - \psi)$ converges to the point (r^*, ψ^*) , which therefore is a node. The $\dot{r} = 0$ and $\dot{\psi} = 0$ loci have a negative slope – a feature that prevailed in all simulations, except for low values of β and ϕ when the slope of the $\dot{r} = 0$ line is positive. Eqs. (17) and (18) provide the intuition about the sign of the slope of the $\dot{r} = 0$ loci. Together they imply:

$$g_r = -\frac{1 - \alpha_1 - \alpha_2}{\alpha_1} \left\{ \left[r - \frac{1 - \gamma}{1 - \alpha_1 - \alpha_2} \alpha_2 \theta (\psi)^{1-\beta} u_P \right] - \beta g_\psi \right\} + (1/\gamma - 1) \frac{\alpha_2}{\alpha_1} g_n,$$

where the term in large brackets is g_w , and the one in square brackets is $\frac{\dot{v}}{v} = r - \pi/v$. As ψ goes up, the productivity of the innovation sector increases. The labor market equilibrium then implies that wages also rise more quickly. A reduction in r would prevent this from happening because it has an adverse effect on the rate of growth of the values of intermediate firms. The slope of the $\dot{r} = 0$ line tells us how much r should drop to prevent innovators productivity from going up.

As for the slope of the $\dot{\psi} = 0$ loci, an inspection of Eq. (13) reveals that if ψ goes up, its growth rate declines. In order to prevent $\dot{\psi}$ from turning negative, either u_E needs to increase or u_I needs to drop or some combination of these two processes must occur. Again, a drop in the interest rate is sufficient because it induces individuals to spend more time in school (since investing in physical capital carries lower rewards) and to pursue more innovation because the future stream of profits is discounted less heavily.

I build the two-dimensional manifold containing the set of solutions of the four dimensional dynamic system following the backward induction technique (Brunner and Strulik (2002)). The basic idea is to make a small step away from the steady state in all possible directions along the linearized stable manifolds and then to integrate the system backward. By doing so, one obtains trajectories of the type depicted in Fig. (1). Clearly, the first step away from the steady state may not be on the actual stable manifold, for we have information only on the linearized stable manifold. Hence, if we try to integrate the system forward, the resulting trajectory will in all likelihood not approach the steady state. However, if the system is integrated backward the trajectory will approach the two-dimensional

stable manifold. By repeating the exercise many times, starting from points picked from a circular grid placed on the stable linearized manifold around the steady state, one can sketch of the shape of actual manifold. The equations are numerically integrated by Matlab 6.5 with the forth-order Runge-Kutta solution method (the error of tolerance is set to 10^{-6}).¹² A snapshot of the trajectories obtained are shown in Fig. (9).

4.1 Productivity Shock

Imagine that the economy is on the steady state and that the final good sector experiences a positive productivity shock (z jumps up). Such a shock indicates that the production process has improved for reasons other than input variety expansion or skill accumulation, where this could be, for instance, process innovation. The greater productivity of physical capital leads to an immediate increment of the interest rate and of labor productivity. In addition, because time is shifted from education into production, the interest rate goes up even further. The productivity shock also boosts the demand for intermediate goods. Thus, time is reallocated from schooling into the adoption of new technologies and production.

Since z does not have an immediate effect on ψ , the economy's position in Fig. (1) is (ψ^*, r') where $r' > r^*$ (the interest rate jump already accounts for adjustment of the choice variables). After the shock, the interest rate declines monotonically towards r^* , whereas ψ follows a U-shaped pattern. The behavior of ψ clearly depends on that of u_E and u_I . Initially u_E declines and u_I rises. Subsequently, however, the quality of education goes up quite rapidly, driven by the expansion of public knowledge, u_E increases, and the fall of ψ is reversed. Fig. (2) shows the time-profile of the four variables. The relationship between initial variations in r, z , and u_P are derived from Eq. (5). One can verify that

$$\hat{z} = (1 - \alpha_2)\hat{r} - (1 - \alpha_1 - \alpha_2)\hat{u}_p,$$

where a $\hat{}$ on the top of a variable denotes percentage deviation from the steady state. In this derivation ψ is kept constant. In sum, the model predicts that the short-run reaction of an advanced economy to a positive technological shock is to introduce new capital goods at a higher frequency, to boost production of final goods, and to reduce schooling time.

4.2 Destruction of Physical Capital

An overnight destruction of physical capital also causes a sudden upward jump of the interest rate. The ratio ψ is not initially affected by the shock. The adjustment process on the $(r - \psi)$ phase-diagram is the same as that just described for the positive technological shock, although some of the underlying mechanisms differ. Labor productivity and wages drop

¹²The Matlab files are available upon request.

immediately after the shock. The time devoted to education may go in either direction. Due to the decline of physical capital, human capital is worth less but at the same time, the opportunity cost of school declines because of the wage reduction. Likewise, two forces of opposite sign affect innovation time. The inward shift of the demand for new intermediate goods causes a compression of the flow of profits. However, innovators do not migrate into the production sector, which also experiences a decline in wages. Indeed, when the simulation is based on the parameter set reported in Table (3) the innovation sector expands. Hence, an overnight decline in the stock of physical capital causes a temporary acceleration of technological progress, weakens the education sector and depresses wages. The adjustment process is still depicted by Fig. (2). Eq. (5) implies that the relationship between initial deviations from the steady state is

$$\hat{k} = \hat{u}_p - \frac{1 - \alpha_2}{1 - \alpha_1 - \alpha_2} \hat{r}.$$

4.3 Inflow of Public Knowledge

The economy so far has been considered closed; therefore, all the public knowledge is generated exclusively from domestic innovation. It is possible, however, for knowledge to percolate from abroad (see Eaton and Kortum (1999) for a discussion). The consequences of an unanticipated one-time inflow of foreign technical knowledge are shown in Fig. (3) (the starting point corresponds to point E' in Fig. (1)). The initial drop of the ratio between human capital and public knowledge sets in motion a number of mechanisms. Labor productivity increases thanks to the availability of a greater variety of tools. Hence more individuals are attracted into the final goods sector, and the interest rate consequently jumps up. Obviously, the productivities of the education and innovation sectors also improve. However, the sudden increase in the number of intermediate inputs reduces the value of newly formed domestic firms. Therefore, the immediate variation of u_I is uncertain, and may actually decline. During the transition, people migrate back into the innovation sector, because the stock market is recovering from the sudden crisis: firms expect that the greater amount of human capital shifts the demand for intermediate inputs and raises the flow of monopoly profits.

4.4 Destruction of Human Capital

In this model, individuals live forever, and there is no depreciation of human capital. Nevertheless, government regulations or social norms may create barriers that limit people's abilities to use their current skills and knowledge. Furthermore, as Galor and Weil (2000) and Galor and Moav (2002) point out, sudden accelerations in n may have a negative effect on

human capital formation because they render existing skills obsolete. Interestingly, a shock that destroys human capital generates a pattern of adjustment similar for the stationary variables to that described for the inflow of public knowledge.

5 Calibration

The objective of this section is to investigate whether the equilibrium dynamics generated by the model are broadly consistent with the development pattern of the U.S. economy since the onset of formal education – about the middle of the 19th century – by means of illustrative calibrations. Standard calibration values will be chosen for the set of preferences and technology parameters $z, \alpha_1, \alpha_2, \gamma, \theta, b, \sigma$, and ρ . Given these parameters, I will run a grid-search in the unit interval for the pair ϕ and β so as to find the combination that delivers time-patterns of key macroeconomic variables that resemble their actual time series counterparts. The constraint $\phi < \beta$ is imposed, for I am interested in a two-dimensional manifold set of solutions converging to the steady state $(r^*, u_P^*, \psi^*, \chi^*)$.

5.1 Data

Education. The fraction of time spent in school, u_E , is estimated by taking the ratio between the average number of schooling years and life expectancy. DHHS (2006) reports average life expectancy in the United States beginning from 1850 for different age groups. For instance the life expectancy in 1850 of 10- and 20-year-old white males were 48 and 40.1, respectively, whereas, in 2000, the corresponding figures were 65.4 and 55.7. On the basis of these data, I estimate a life-span for the representative individual to be about 59 (the average between 58 and 60.1) for the year 1850 and 75.5 for the year 2000. For the 19th century, DHHS (2006) provides only two data points: 1850 and 1890. From 1900 on, the frequency is at least every 10-year interval. As concerns the duration of schooling, I rely on the recent estimates elaborated by Baier et. al. (2007). This study calculates the average years of schooling of the labor force in the U.S. on a 20-year interval, starting from 1840. Because the starting point and the frequencies of the two series do not coincide, some interpolation was needed to fill missing observations for the 19th century. Table (5) shows a snapshot of the data and of my estimates for u_E .

Output. The output of final goods, y , is matched with the per capita GDP time series elaborated by Maddison (2003). For the United States, this is available on an annual basis from 1870 onward, whereas, for the earlier part of the 19th century, it is available on a 10-year interval basis.

Interest Rate. Table IV of Barro (2006) shows that the real stock return in the United

States for the 1880-2004 and 1954-2004 time periods are 0.081 and 0.089 respectively. The real bill return for the same two periods is 0.015 and 0.017, respectively. Siegel (1998) calculates a return of 7% for the periods 1802-1997, 1871-1997 and of 6.7% for the period 1913-1997 (see Table 8-1). Hence, I will try to calibrate the model so that during the transition the real interest rate is somewhere between these rates and roughly constant.

Consumption-Capital Ratio. Consumption and capital stock time series are drawn from the National Income and Product Accounts (NIPA) and from the Fixed Assets data. Both series are available from 1925 up to 2007 at the Bureau of Economic Analysis.¹³ The consumption-capital ratio, χ , is matched with the ratio between non-durable consumption expenditures and the (net-cost) value of fixed assets. Both series are in current dollars.

Output-Capital Ratio. The denominator of the ratio is still fixed assets. The numerator is the NIPA GDP.

Total Factor Productivity. The plausibility of the model will also be judged against some simple growth accounting. Gordon (2000, Table 1) calculates the growth rate of total factor productivity, GDP, capital, and labor for the U.S. economy from 1870 to 1996. Starting from Gordon's estimates, I compute the share of per capita output growth accounted for by total factor productivity and compare it with the corresponding ratio implied by the model (see Table (5))

5.2 Parameters

My model economy is fully characterized by 9 parameters, $z, \alpha_1, \alpha_2, \gamma, \theta, b, \phi, \beta, \sigma$, and ρ , and two initial values $r(0)$, and $\psi(0)$. I choose the parameters as follows. First, I adopt the standard parameter value for the discount rate ($\rho = 0.02$) for the inverse of the intertemporal elasticity of substitution ($\sigma = 2$) and for the labor share ($\frac{1-\alpha_1-\alpha_2}{1-\alpha_2}$) which is set to 0.7. The baseline value of α_1 is 0.162. This value allows the model economy along the transition to match a 5% interest rate – which I pick as my target interest rate – and an output-capital ratio of about 0.33, which is observed in the data. The implied value for α_2 is 0.46.¹⁴ The productivity parameter of the education sector $b = 0.55$ – roughly the same as the one used by Lucas (1988). The innovation rate is strongly related to both θ and γ . Hence, I fix $\gamma = 0.6$ (the elasticity across intermediate goods) and then choose θ so that, given the other parameters, the balanced growth path of income is 1.5% – slightly smaller than the 1.8%

¹³See NIPA Table 1.1.5 "Gross Domestic Product", and Table 1.1. "Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods" that can be extracted from the Bureau of Economic Analysis data set available at <http://www.bea.gov>.

¹⁴These values are similar to those used in previous studies. See, for example, Acemoglu and Guerrieri (2008), Kongsamut et al. (2001), Eicher and Turnovsky (2001), Funke and Strulik (2000), and Ortigueira and Santos (1997).

secular U.S. per capita output rate of growth, which is here interpreted as a transitional phenomenon.

Two important parameters for my calibration are β and ϕ .¹⁵ I will try to find the pair of values that is most likely to characterize the U.S. economy by matching the transitional dynamics of the model with the U.S. economic data in the last 150 years. From a grid search, whose details are discussed in section (5.4), it turns out that the pair $\phi = 0.4$ and $\beta = 0.5$ gives a very good fit.

5.3 Results

Row (1) of Table (3) summarizes my preferred values for the 9 parameters. These are used in the benchmark calibration. Column (c) of Table (7) shows the value of seven macrovariables on the BGP. Fig. (4) depicts the time trajectories along the transitional dynamics of six of these variables: the share of education time (u_E), capital-consumption ratio (χ), capital-output ratio (y/k_1), the interest rate (r), per capita output (in logs), and the ratio of total factor productivity (TFP) growth to output growth for about 150 years starting from the middle of the 19th century. Two measures of TFP are used: one considers only the contribution of technological progress, while the other includes the contribution of both education and innovation. The simulated pattern of each variable (dashed line) is compared against the U.S. time series (continuous line), when this is available.

From a visual inspection of Fig (4) a key aspect emerges: the schooling time rises from zero to about 18 percent and yet per capita output growth, the interest rate, the output-capital ratio, and the consumption-capital ratio remain roughly constant. Because of the law of diminishing returns on physical capital, the transitional dynamics of neoclassical growth model are characterized by a marked decline in the interest rate (see King and Rebelo, 1993) and a deceleration in output growth. If human capital formation is added to the neoclassical model, the decline of the returns on physical capital is instrumental in triggering investment in education.¹⁶ In the model presented here, individuals spend more time in school not because investing in physical capital becomes less profitable, but because the return on

¹⁵Charles Jones' work and the non-scale innovation-based growth models suggest that β should be less than one (see Jones, 2005, section 5). As concerns the educational sector, the standard Mincer specification for human capital formation has neither externalities from public knowledge nor any positive effects from the existing stock of human capital of the dynasty. Lucas (1988) assumes a strong externality for human capital (\dot{h} is linear in h) but public knowledge is neglected. Bils and Klenow (2000) speculate that the human capital externality is much smaller than one, but again do not include public knowledge externalities.

¹⁶Interesting exceptions are Acemoglu and Guerrieri (2008) and Kongsamut et al. (2001). However, they analyze the unbalanced growth of two final goods sector. Here, in contrast the constancy of the growth rate of output and the relatively little variation of the interest rate are features of the transition towards the balanced growth path.

education rises thanks to the growing stock of public knowledge. This is the main reason the transition occurs with a nearly constant interest rate, which oscillates between 5 and 5.2 percent. The model also does a remarkable job at capturing the level and time trend of all key variables. The per capita output grows at the same steady pace as the actual one, and education time closely parallels the secular upward trend of the time series. Both the capital-output ratio and the consumption-capital ratio are also consistent with the actual time series that starts from 1929. Unfortunately, the empirical counterpart for ψ and u_I is missing. The consistency of their time-patterns is inferred indirectly by means of growth accounting. The basic idea is to check whether the contribution of per capita output growth attributable to the Solow residual matches with the empirical one. The piecewise line in Panel C (see also Table 6) shows that, for most of the time periods considered, the contribution of the residual to output growth calculated with U.S. long-run data fluctuates between 0.4 and 0.9. A peak is recorded in the 1928-1950 time interval (1.4) and an exceptional low ratio (0.26) is displayed for the 1970s. The simulated trajectory stays fairly constant at about 0.8.

A final characteristic deserves attention: the lack of acceleration of per capita output despite the rapid expansion of human capital. There are two forces that put a brake on such acceleration. One is the decline in the fraction of production time, which is diverted into schooling;¹⁷ the other is the gradual decline of time allocated to innovation that causes a slow down in technological progress. The substitution of innovation for education is illustrated in Fig (4) Panel C. The fraction of labor productivity growth accounted for by innovation ($\frac{(1/\gamma-1)\alpha_2}{1-\alpha_2}g_n/(g_y - g_u)$), declines over time (bottom dashed line), whereas the joint contribution of innovation and education ($[\frac{1-\alpha_1-\alpha_2}{1-\alpha_2}g_h + \frac{(1/\gamma-1)\alpha_2}{1-\alpha_2}g_n]/(g_y - g_u)$) is roughly constant. Hence, a 'replacement effect' is taking place: less time is devoted to the adoption of new technologies, and more time is spent acquiring skills. Although this conjecture cannot be verified directly, it is consistent with the trend-less behavior of the ratio between U.S. total factor product growth and output growth, displayed in Panel C.

6 Discussion

In this section, I want to highlight one aspect of the transitional dynamics: The onset of the education sector occurs only after the set of public knowledge has reached a certain threshold. This observation is important, for it gives an historical explanation for the rise of education in today's advanced economies, and it provides a suggestive argument for why developing countries lagged considerably in carrying out education reforms. The analysis of the transitional dynamics of Uzawa-Lucas model does not have an historical content, in the

¹⁷This effect would not exist if the value added generated by the education sector were included in the computation of output.

sense that the conditions for the onset of the education sector could have been present at any time. The equations representing the equilibrium for a Lucas-Uzawa type of economy are (15), (17), and (19), with g_w determined through (21), and with $u_I = g_n = 0$, and with $\phi = 0$. The key condition for the arrival of formal education is that the inequality $r > g_w + b$ turns into an equality. Such an event may occur either because the interest rate declines, or because there is an acceleration in the real wages or an improvement in the individual's learning capacity. As was discussed in the introduction, the interest rate does not seem to have declined in the last two centuries. Although an acceleration of the wage rate prior to the onset of formal education is broadly consistent with historical records, wage accelerations did also occur in the Middle Ages (Clark (2005)) but did not have any effect on education. An exogenous positive shock hitting b is also quite an ad-hoc explanation: why did such a shock not occur in Ancient Rome? Additionally, research on evolution does not indicate any significant change in human intellectual capacity in the last few millennia.

The dynamics of my model economy suggests that when the set of information available hits a certain threshold, education becomes a profitable investment. The first noticeable modern form of education in technical and scientific fields occurred at the turn of end of the 18th century when technological progress took a turn into the areas of chemicals and electricity. The first modern schools where students learned applied science and technology were established in France.¹⁸ This sparked emulation in the many European urban centers (Zurich, Prague, Vienna, and places as far off as Moscow) and later on in the U.S. as well. However, an enormous amount of technological progress occurred well before the industrial revolution.¹⁹ Variants of Leonardo's hydraulic, work, and war machines had already been in use in the twelfth and thirteenth century, and some were known – although used only for minor production activities – in Ancient Rome. Yet, economies have functioned with virtually no formal education until about 150 years ago, despite technological advances over the whole of human history. It was only when the variety of scientific and technical fields reached a critical mass that societies started to discuss educational reforms. As White (1962, p. 129) and Landes (1998, p. 283) point out, until the arrival of electricity and chemicals people were mostly engaged in elaborating and refining principles established during the four centuries before Leonardo.

¹⁸The Ecole Polytechnique was founded in France in 1794. This soon turned into a center for math and basic science. Subsequently, however, a number of pioneering schools with a focus on technical subject were established, including the Ecole de Mines, the Ponts-et-Chaussees, and the Ecole Central des Arts et Manufactures.

¹⁹See among others Cipolla (1965), and Diamond (1997).

6.1 Bridging two stages of development

Formally, the onset of education occurs when Eq. (10a) replaces (10b). An illustration of the transition from one development stage to the other is provided by Fig. (7) and Fig. (8). They represent an extension of the benchmark economy to include a development stage where innovation is the only engine of growth. Briefly, the procedure consists of integrating backward the four-dimension system of the benchmark economy until u_E approaches zero. At that point, the equilibrium dynamics of the economy with no schooling, namely the innovation-only economy, takes over (Eq. (10b) replaces Eq. (10a)). The initial point of the backward integration of the innovation-only economy is to find the value of the three-dimensional vector (r, ψ, u_P) recorded when u_E is an epsilon away from zero.²⁰ The set of parameters values are the same in both development stages, except that θ is reduced in the innovation-only economy in order to match the slower pace of income growth that characterized the U.S. economy in the century and a half prior to the onset of formal education (approximately from 1700 to 1850). In the innovation-only economy, the interest rate declines by almost one percentage point, whereas it is roughly flat in the two-engine economy. Clearly, when investment in human capital is not feasible, physical capital exerts a greater role in driving the transition. Another point to highlight is that the innovation-only economy asymptotically tends to stagnation as long as $\beta < 1$. This outcome is prevented by the arrival of formal schooling: The rising level of human capital in the innovation sector compensates for the diminishing marginal contribution of the stock of public knowledge.

As long as the model has a two-dimensional manifold of equilibrium solutions, the critical threshold is not unique. It depends on the prevailing value of the interest rate. This property of the dynamics is clarified by Fig (10), which depicts four trajectories in the phase-space,²¹ all of which show an increase in education time as the economy approaches the steady state (in terms of Fig. (1), the initial position is somewhere in region I or II). Notice that the threshold is inversely related to the interest rate. More specifically, this means that the waiting time for the onset of education is longer in three types of places: i) where the reward to capital is high – thus, individuals are more demanding in terms of the returns they require from school; ii) where the workers' endowment of human capital is high and therefore the opportunity cost of going to school in terms of missed wages is high; iii) where there is a shortage of public knowledge, and the quality of the school sector consequently is low.

Property (i) is consistent with human capital growth models that imply that it was the

²⁰For more details on this technique, see Brunner and Strulik (2002).

²¹The values of the underlying parameters are identical to the ones used for the previous calibration illustration (Table (3)), and one of them – curve C – corresponds to the calibrated time-trajectories shown in Fig. (4).

high interest rates that kept education at bay. In contrast, the current model argues that this is only one macroeconomic reason for delays educational reforms. Property (ii) is at odds with the studies that emphasize the scarcity of initial human capital as the key reason for the delayed transition into a modern stage of development (see for instance Becker et al. (1990)). Interestingly, there is at least one historical event that conforms well with my model’s prediction: in the 18th century UK workers were more skilled than their continental counterpart, and yet the onset of the UK educational system came considerably later than those of other European countries. At the at the end of the 19th century UK was behind France, Germany, and the US in formal education.²² Indeed, the UK largely followed a learning-by-doing development strategy with little reliance on education until the emergence of chemicals and electricity (see Landes (1998, Ch. 18)). Property (iii) is a quantitative restatement of the idea that public knowledge must reach a critical threshold for education to set in. Given the interest rate and the stock of human capital, it establishes the minimum level of public knowledge that is needed to induce individuals to spend time in school.

Another implication of Fig (10) is that countries that share the same underlying parameters converge to the same steady state starting from different initial conditions, but that they follow different routes. Fig (9) plots the time patterns of the economy calibrated in the previous section, and those of two alternative economies that share the same parameter values, the same per capita output in 1840, but have different initial conditions for r and ψ . The figure shows that the three economies follow three different routes, and that the gap of the interest rate, consumption-output ratio, TFP ratio declines along the transition. The growth rate of per capita output also converges, although difference in levels may persist on the balanced growth path.

7 Sensitivity Analysis

How do alternative specifications of the ϕ and β affect the equilibrium? From this section, it will emerge that although a wide range of values of ϕ and β generates similar balanced growth path equilibria, the transitional dynamics patterns are quite sensitive to alterations of the two parameters.

In a first experiment (E1) β is increased by 0.1 increments starting from the benchmark value of 0.5. In a second experiment (E2) ϕ is lowered by 0.1 at a time beginning from the 0.4 baseline value. In the third experiment (E3) ϕ and β are moved in lock-step within the unit interval, keeping a distance between them of 0.1 – as in the benchmark economy. In all the experiments, the remaining parameters are unaltered, except for θ , which is adjusted to

²²Easterlin (1981) estimates that, in the year 1900, the primary school enrollment rates in the UK, France, Germany and USA were 0.1407, 0.1412, 0.1576, 0.1969, respectively.

have a BGP interest rate and an output growth rate at 5% and 1.5%, respectively.

7.1 Balanced Growth Path

Panel A and B of Table (8) summarizes the outcome of E1 and E2, respectively, when the economy is on its BGP. In the set of experiments E1, θ needs to be moved in the same direction as β : because the benchmark economy is human capital intensive vis-à-vis public knowledge ($\psi > 1$), a positive change in β would otherwise lead to a decline in the innovation rate. An appropriate increment of θ prevents that from happening. Similarly, in E2 experiments, ϕ as drops the growth rate of human capital shoots up and a reduction in θ is needed to slow down the process of innovation.

Of the seven key variables considered only ψ changes significantly across experiment E1: as β goes up, the BGP economy is less human capital intensive. In E2, ψ is more sensitive to ϕ . For instance, when this goes from 0.4 to zero, ψ rises about 5 times. This huge human capital increment reduces the required schooling time needed to sustain a given growth rate in human capital: u_E goes from 17% to 10%.

In E3 (Table (9)) the elasticity of public knowledge is changed in *both* knowledge sectors at the same time. A positive change of β and ϕ must be compensated for an upward shift of θ – otherwise the growth rate of output would be too low due to the diminished contribution of human capital in both knowledge sectors. The rise in θ brings u_E and u_I down by a small margin, but the remaining 5 key BGP variables appear nearly identical as those of the BGP. The insensitivity of the BGP to variations of the elasticity parameters also holds for alternative specification of the labor share (0.6, 0.65, and 0.75).

In brief, the comparative dynamics experiments reveal that economies with different knowledge-elasticities are quite similar on the BGP.

7.2 Transitional Dynamics

Safe for the benchmark economy, the dynamics of economies in E2 (Panel (B)) are characterized by two complex stable eigenvalues with identical (negative) real part. This means that the system converges to the steady state in an oscillating manner along a one-dimensional stable manifold – one dimension short of the benchmark saddle-path dynamics. More interesting are the E1 economies, which exhibit a two-dimensional manifold set of equilibria. As β increases from the 0.5 benchmark value, the time-pattern of education and that of output converge more quickly than the actual U.S series. The innovation sector of the high- β economy expands more rapidly than that of the low- β economy, because public knowledge carries a greater role an externality. Furthermore, the acceleration in n boosts the efficiency of the education sector relatively more in the high- β economy. Consequently, individuals increase

their schooling time more quickly in an economy with a higher β , implying that the economy converges more quickly towards the BGP. Fig. (5) shows one such an example ($\beta = 0.9$): it takes less than ten years for u_E to go from 0 to 0.1. As a result, the generated logs of the output pattern is visibly more curved than the actual one.

In addition, the transitional dynamics associated with E3 are two-dimensional saddle-path stable around the steady state. Although they do not match the U.S. time series, some of them are quite plausible and could mimic the development trajectories of other countries. For instance, when $\phi = 0.5$ and $\beta = 0.6$, education and output follow a similar pattern as in the benchmark economy, except that transition is slower (Fig (6)).

In sum, economies that differ substantially with respect to ϕ and β look similar to each other on the steady state. The transitional dynamics, however, indicate that the U.S. economic history of the 150 years is best explained when the distance between β and ϕ is about 0.1 and when their values are in the middle of the unit interval rather than at the extremes.

8 Conclusion

The main conjecture was that the expansion of public knowledge created the historical conditions for the onset and rise of formal education in modern societies. As the variety of technology expands, new opportunities to acquire knowledge appear and the expected returns on education increase, inducing individuals to spend a larger fraction of their lives to acquire human capital. As with prior studies, I see education as a form of investment that compete with physical capital formation, but my emphasis is on the increased efficiency of the education sector driven by the new windows of public knowledge, rather than on the declining marginal productivity of physical capital. I investigated the implications of these hypotheses by studying the transitional dynamics of a growth model with two complementary sources of long run growth (innovation and education), and verified their plausibility through calibration analysis. The transitional dynamics generated trajectories in which the interest rate and other key macroeconomic ratios remained flat despite the remarkable rise of education, bringing additional insights on how to reconcile Kaldor facts with fundamental structural changes. The dynamics of the model indicated that production time is only marginally affected by the rise of education, for most of the schooling time is subtracted from entrepreneurial activities. Hence, as economies mature, more effort is devoted to exploit the stock of knowledge for productive purposes and less to expand it.

The paper leaves important questions open, some of which can only be assessed empirically. One is the extent to which public knowledge can trade-off the high returns of physical capital. The literature indicates remarkably high rates of returns on specific capital goods in

developing countries.²³ High returns do not seem to have kept education at bay altogether, but they may limit its expansion. A policy that promotes faster accumulation of physical capital would accelerate the transition towards higher levels of education. However, this may be a non-viable path, especially if the majority of the population is close to a minimum level of consumption. A faster alternative suggested by the model is the rapid expansion of public knowledge through the adoption of foreign technologies, as long as the knowledge that these technologies embody can be studied; that is, it can be transformed into human capital.

A related aspect that would need further examination is the set of conditions that would allow the education sector to have the greatest possible access to public knowledge. The model did not make a distinction between what is known and what is taught in schools. Presumably, only a fraction of what is known is subject to systematic teaching in schools. However higher educational institutions seem to compete fiercely with each other by offering new curricula that impart instructions in emerging fields or at the intersection of existing fields. The outcome of such competition has important dynamic consequences: The great expansion of schools in computer science in the 1980s and 1990s is an example of educational innovation inspired by the dissemination of information technology. Similarly, the financial innovation of the late 1980s was followed by a proliferation of programs in finance. Arguably, the former type of educational innovation is more likely to bring long-run benefits than the latter one. Conversely, in pre-modern societies, many barriers were erected to keep innovations secret, either out of protectionism or as an attempt by the ruling government to preserve the political power.²⁴ The scientific revolution of the seventeenth century, arguably, accelerated the process of transforming knowledge into a public good, and it may have shortened the waiting time for the onset of formal education. Mokyr (2005), for instance, highlights the proliferation of scientific and trade organizations, and the systematic collection of 'useful' knowledge into encyclopedias at the turn of the 18th century as two clear signs that knowledge was increasingly nonproprietary and that discoveries were becoming more like public goods. Finally, the requirement that the patent blueprint must be disclosed to the public may have also facilitated the dissemination of knowledge through formal education.

²³Udry and Anagol (2006), for instance, estimate that the return to capital in Ghana's informal sector is 60%. Double-digit returns have been found in numerous other papers that focus on developing countries. For a summary see Banerjee and Duflo (2005, p. 479-484).

²⁴In the 14th century Venetian glassmakers were not allowed to leave the Republic. Those who did faced the sanction of the death penalty.

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Appendix A

Eqs. (13), (15), (17), (19), and (22) evaluated at the balance growth path reduce to

$$0 = bu_E(\psi)^{-\phi} - \theta u_I(\psi)^{1-\beta} \quad (\text{A1})$$

$$0 = \left(\frac{1}{\sigma} - A_5\right)r + \chi - \frac{\rho}{\sigma} \quad (\text{A2})$$

$$0 = A_1[r - A_3\theta(\psi)^{1-\beta}u_P] + A_2\theta u_I(\psi)^{1-\beta} \quad (\text{A3})$$

$$0 = A_4[r - A_3\theta(\psi)^{1-\beta}u_P] + (A_2 - 1)\theta u_I(\psi)^{1-\beta} + A_5r - \chi. \quad (\text{A4})$$

and

$$0 = b(\psi)^{-\phi}(1 - \phi u_E - \beta u_I) - A_3\theta(\psi)^{1-\beta}u_P \quad (\text{A5})$$

where $A_1 \equiv -\frac{1-\alpha_1-\alpha_2}{\alpha_1}$, $A_2 \equiv (1/\gamma - 1)\frac{\alpha_2}{\alpha_1}$, $A_3 \equiv \frac{1-\gamma}{1-\alpha_1-\alpha_2}\alpha_2$, $A_4 \equiv -(\frac{1-\alpha_2}{\alpha_1})$, $A_5 \equiv \frac{1-\gamma\alpha_2}{\alpha_1}$ and where I have already used Eq. (18) to replace g_w in Eq. (17) and (19). In addition, (1) must hold. Hence we have a system of six equations in six unknown: u_P, u_E, u_I, r, χ , and ψ .

The variable χ from Eq. (A4) is eliminated through (A2) and the resulting equation is combined with (A3) to eliminate r . Then, Eq. (A4), after rearrangements, becomes

$$u_P = \frac{\rho}{c_2\sigma\theta(\psi)^{1-\beta}} - \frac{c_1}{c_2}u_I, \quad (\text{A6})$$

where $c_1 = [(A_2 - 1) - (A_4 + \frac{1}{\sigma})\frac{A_2}{A_1}]$ and $c_2 = [(A_4 + \frac{1}{\sigma})\frac{A_3}{A_1} - A_4A_3]$.

Plugging (A1) into (1) this becomes.

$$u_P = 1 - \left[\frac{\theta}{b}(\psi)^{1-\beta+\phi} + 1\right]u_I. \quad (\text{A7})$$

Eq. (A1) can also be used to eliminate u_E from Eq. (A5), which yields the third needed function linking u_P , to u_I and ψ . However, the resulting relationship is quite convoluted. It is easier to set two conditions for the existence of an equilibrium only on the basis of Eqs. (A6) and (A7): (i) $\frac{c_1}{c_2} < 0$ and (ii) $\frac{\rho}{c_2\sigma\theta(\psi)^{1-\beta}} < 1$. The latter is equivalent to $\frac{\rho}{c_2\sigma} < g_n/u_I$.

Appendix B: Special Cases

In this section I want to solve the model for the competitive equilibrium in three special cases in which β and ϕ are either 0 or 1.²⁵ Before proceeding it is useful to define the following parameters: $B_1 \equiv \frac{1-\alpha_1-\alpha_2}{1-\alpha_2}$, $B_2 \equiv (1/\gamma - 1)\frac{\alpha_2}{\alpha_1}$, $B_3 \equiv \frac{1-\gamma\alpha_2}{\alpha_1}$, $B_4 \equiv [\alpha_2(1 - \gamma)/(1 - \alpha_1 - \alpha_2) - b/\theta]$, and $B_5 \equiv (B_2\theta - bB_1)$.

²⁵The combination $\beta = 0$ and $\phi = 1$ is neglected because is analytically intractable.

Case 1: $\beta = \phi = 0$. The education and innovation technology is $\dot{h} = bu_E h$, and $\dot{n} = \theta u_I h$. Neither the education sector nor the innovation sector benefits from any externality arising from non-rival knowledge, but human capital is used in all three sectors. It combines the Grossman-Helpman model with that of Uzawa-Lucas. Funke and Strulik (2000) show the dynamic properties and the policy implications of a similar model. The evolution of r and χ in Eqs. (17) and (15) become

$$g_r = -B_1(r - b) + B_2 g_n \quad (\text{B1})$$

$$g_\chi = \left(\frac{1}{\sigma} - B_3\right)r + \chi - \frac{\rho}{\sigma}, \quad (\text{B2})$$

whereas relationship (22) simplifies to

$$b = \frac{\alpha_2}{\alpha_1} \frac{1 - \gamma}{B_1} \theta \psi u_P, \quad (\text{B3})$$

implying that u_P and ψ move in lock-step not only on the balanced growth path but also during the transitional dynamics. Since $g_{u_P} + g_\psi = 0$, the sum of the right-hand sides of Eqs. (19) and (13) should also be equal to zero. From this operation one finds that g_n depends only on r and χ :

$$g_n = [\gamma B_2 r + b \frac{1 - \alpha_2}{\alpha_1} - \chi] / (1 - B_2). \quad (\text{B4})$$

Inserting (B4) into (B1) and (15) one finds out that the g_r and g_χ equations form a subsystem that is independent of ψ :

$$\begin{aligned} g_r &= -B_1(r - b) + \frac{B_2}{1 - B_2} (\gamma B_2 r + b \frac{1 - \alpha_2}{\alpha_1} - \chi) \\ g_\chi &= \left(\frac{1}{\sigma} - B_3\right)r + \chi - \frac{\rho}{\sigma}, \end{aligned}$$

and that

$$g_\psi = b \left\{ 1 - \frac{1}{\theta \psi} \left[\frac{B_1 \alpha_1}{\alpha_2 (1 - \gamma)} + g_n \right] \right\} - g_n.$$

The two-equation subsystem $g_r - g_\chi$ generates a one-dimensional saddle-path in the space (r, χ) if $(\frac{1}{\sigma} - \frac{1}{\alpha_1} + \gamma) \frac{B_2}{1 - B_2} < B_1$: under this restriction the determinant of the Jacobian of the linearized system around the steady state is negative, implying that there exists one positive and one negative eigenvalue. The Jacobian of the three-equation system (r, χ, ψ) linearized around the steady state has one negative and two positive eigenvalues for a wide combination of parameters that deliver steady state values close to those commonly observed in advanced economies. This means that for a given r_0 , there exists only one pair (χ_0, ψ_0) that ensures that the economy undertakes the trajectory leading towards the balanced growth path.

Case 2: $\phi = 0$ and $\beta = 1$. Public knowledge generates spillovers in the innovation sector, but not in the education sector, and the ability to innovate does not increase with education. The resulting

model keeps Uzawa-Lucas features, whereas the innovation sector works as in Romer (1990). I will refer to it as the RUL case. There is no direct link between the innovation and education sectors. Innovators learn from past discoveries, but the ability to innovate does not increase with education. Lloyd-Ellis and Robins (2002) employ a similar specification, except that the stock of human capital enters non-linearly in the education production function. Following condition (22), we have

$$u_I = B_4 u_P.$$

The three key differential equations of the model are

$$g_r = -B_1(r - b) + B_5 u_P,$$

$$g_{u_P} = [B_3 - (B_1 + 1)]r + bB_1 + (b + B_5)u_P - \chi,$$

and

$$g_\chi = -\frac{\rho}{\sigma} + \left(\frac{1}{\sigma} - B_3\right)r + \chi.$$

One can verify that the steady state interest rate is $r^* = [\frac{\rho}{\sigma} + \frac{b}{B_5}B_1b]/[\frac{1}{\sigma} - (1 + B_1) + (1 + \frac{b}{B_5})B_1]$, and that the Jacobian of the linearized system around the steady state is

$$J = \begin{bmatrix} -B_1 r^* & B_5 u_P^* & 0 \\ [B_3 - (B_1 + 1)]r^* & (b + B_5)u_P^* & -\chi^* \\ (\frac{1}{\sigma} - B_3)r^* & 0 & \chi^* \end{bmatrix}.$$

Its determinant is negative if and only if $B_5(1 - \frac{1}{\sigma}) < B_1 b$. This condition taken together with a positive trace for the Jacobian ensures that the economy converges to the steady state with a saddle-path dynamics: given an initial r_0 there is one and only one initial pair (u_P, χ) that puts the economy on the stable manifold.

Whereas case 1 required a specific initial value ψ_0 for a given r_0 , in this economy the initial condition of ψ is irrelevant for the transitional dynamics. If the economy starts from an initial position in which the interest rate is above the steady state interest rate, the transition is characterized by a gradual expansion of the education sector and a contraction of the innovation sector. If the interest rate is high enough there is no investment in human capital at all and growth is driven by capital good expansion.

Case 3: $\phi = \beta = 1$. The stock of knowledge produces externalities in both the innovation and learning sector. There is no intergenerational transfer of human capital: it is not what the parents know, but rather the advances of the technological frontier that increases the offsprings' ability to learn. Along the transitional dynamics the knowledge-human capital ratio remains constant ($\psi = \frac{b}{\theta} / B_6$, where $B_6 \equiv \frac{(1-\gamma)\alpha_2}{1-\alpha_1-\alpha_2}$ - see Eq. 22). Consequently, $g_\psi = 0$, which implies that $u_I = u_E B_6$ (see Eq. 13). In addition to these two equations, the dynamic system is described by

$$g_r = -B_1(r - B_6\theta u_p) + B_2\theta(1 - u_p)/(1 + 1/B_6),$$

$$g_{u_p} = -(B_1 + 1 - B_3)r + (B_6 - 1)\theta(1 - u_p)/(1 + 1/B_6) - \chi,$$

$$g_\chi = \left(\frac{1}{\sigma} - B_3\right)r + \chi - \frac{\rho}{\sigma}.$$

After some algebra, one finds that $u_I^* = \frac{B_6+1/(1-1/\sigma)(\rho/(\theta\sigma)-B_6)}{1+B_6(1-1/\sigma)-(2+B_6)/(1-1/\sigma)}$, and $r^* = B_6\theta - (B_6 + 2)\theta u_I^* - \rho/\sigma$. The remaining steady state values are easily derivable from the above relationships. The determinant of the associated Jacobian is negative if $B_6 < \frac{B_2}{B_6+1} < 1 < \sigma B_1$. To have a saddle-path dynamics pattern, the Jacobian also needs a positive trace (i.e. $\chi^* > B_1 r^* + (B_6 - 1)\theta u_p^*/(1 + 1/B_6)$). Scenarios in which the system is unstable (three positive eigenvalues) or stable (two or three negative eigenvalues) are not interesting from an historical point of view; therefore they will be examined in greater detail. The main drawback of the dynamics obtained in this case is the lock-step movement of education and innovation: the two sectors are predicted to set in at the same time, which restricts the ability of the model to account for diverse development trajectories.

Table 3: Baseline Parameters

	z	α_1	α_2	γ	θ	b	ϕ	β	σ	ρ
(1)	1	0.162	0.46	0.6	0.1	0.055	0.4	0.5	2	0.02
(2)	1	0.2	0.2	0.6	0.5	0.05	0.2	0.3	2	0.02

Table 4: Comparative Dynamics

	σ	ρ	θ	b	ϕ	β	α_1	α_2	γ
u_E	-	-	+	+	-	-	\cap	\cap	\cap
u_I	-	-	+	+	-	-	\cap	+	-
ψ	-	-	-	+	-	+	+	-	+
χ	+	+	+	+	-	-	-	+	-
r	+	+	+	+	-	-	-	+	-
g_y	-	-	+	+	-	-	-	+	-

Note. The + (-) sign indicates that the variable evaluated at the steady state is increasing (decreasing) in the parameter. The sign \cap denotes an inverted-U relationship. The model is parametrized with the set of values reported in the first row of table (3).

Table 5: Fraction of Time Spent to Acquire Human Capital

Time	1840	1900	1940	1960	1980	2000
Years of school	1.14	4.83	8.28	9.83	11.7	13
Life span	58.92	61.39	67.39	70.01	72.21	75.55
u_E (estimate)	0.012	0.079	0.123	0.1404	0.1620	0.1721

Note. – The first row reports the average years of schooling of the labor force estimated in Baier et al. (2007, Table 1). The life span is calculated on the basis of estimates of life expectancy of 10 and 20 year-old white male provided by the DHHS (2006). The life expectancy in 1840 and 1900 is interpolated from observations of the year 1850 and 1890. The fourth row reports the ratio between the first and the second row.

Table 6: Ratio between the rate of growth of TFP and that of per capita GDP

Time	1870-91	1891-1913	1913-28	1928-50	1950-64	1964-72	1972-79	1979-88	1988-96
Ratio	0.46	0.75	0.84	1.42	0.70	0.49	0.26	0.40	0.79

Note. – The growth accounting is based on Gordon (2000, Table 1). The production factors are not adjusted for quality.

Table 7: Calibration on the Balanced Growth Path

		(a)	(b)	(c)	(d)
		s_L 0.6	0.65	0.7	0.75
	U.S. Data	θ 0.0425	0.0645	0.1	0.1886
r	0.013-0.07	0.05	0.0502	0.05	0.0503
y/k_1	0.33	0.3086	0.3099	0.3086	0.3105
g_y	1.8	0.015	0.0151	0.015	0.0151
ψ	NA	1.6185	1.4748	1.3838	1.2266
χ	0.21	0.1834	0.1948	0.2086	0.2297
u_E	0.16	0.1265	0.1619	0.1739	0.2015
u_I	NA	0.1053	0.0879	0.0714	0.0489

Note. – The table compares the values of the seven key model’s variables evaluated at the BGP against the corresponding U.S. data – when available. In column (c) the model is evaluated under the set of parameters displayed in Table (3), row (1). Column (a), (b), and (d) consider three alternative values for the output elasticity to skilled labor. In each case θ is adjusted to maintain the interest rate at 5% and g_y at 1.5%. The data values are approximate averages over the length of the available time series, except for u_E , which refers to the year 2,000.

Table 8: Sensitivity Analysis: I

Panel A: $\phi=0.4$					
β	0.6	0.7	0.8	0.9	1
θ	0.1038	0.1088	0.11	0.115	0.1175
r	0.0501	0.0504	0.0498	0.0504	0.0504
y/k_1	0.3093	0.3111	0.3074	0.3111	0.3111
g_y	0.0150	0.0152	0.0149	0.0152	0.0152
ψ	1.3557	1.3091	1.3206	1.2566	1.2319
χ	0.2087	0.2099	0.2077	0.2102	0.2102
u_E	0.1697	0.1697	0.1673	0.1670	0.1658
u_I	0.0708	0.0711	0.0708	0.0714	0.0714
Panel B: $\beta=0.5$					
ϕ	0	0.1	0.2	0.3	0.4
θ	0.41	0.0425	0.0788	0.0938	0.1
r	0.0498	0.0497	0.0501	0.0499	0.0500
y/k_1	0.3074	0.3068	0.3093	0.3080	0.3086
g_y	0.0149	0.0148	0.0151	0.0149	0.0150
ψ	6.9321	6.9374	2.2665	1.6269	1.3838
χ	0.2077	0.2072	0.2091	0.2080	0.2086
u_E	0.1031	0.1370	0.1778	0.1889	0.1739
u_I	0.0762	0.0732	0.0702	0.0690	0.0714

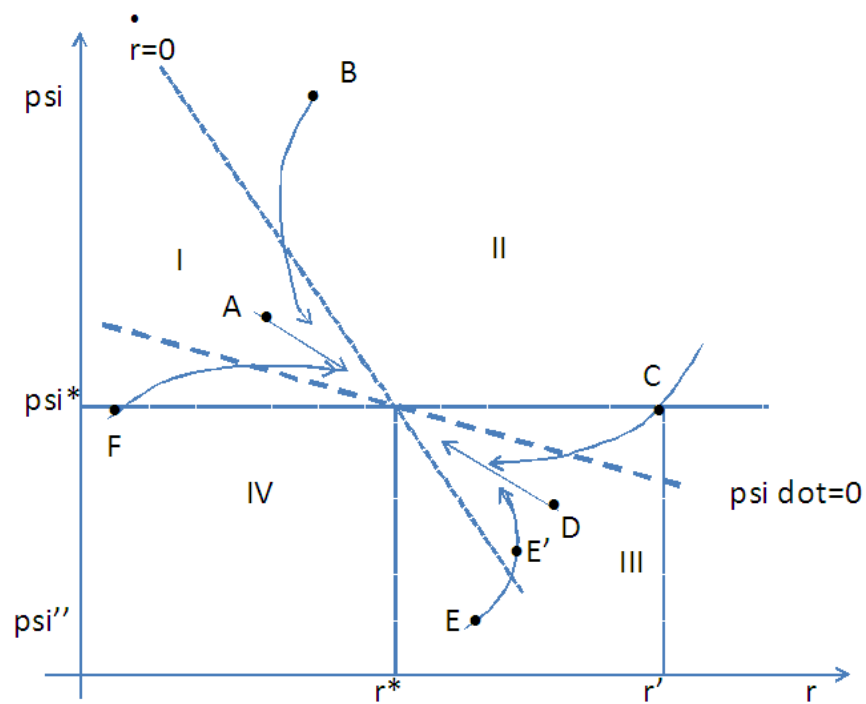
Note. – Sensitivity analysis of economy of Table 5, column C (labor share $s_L = 0.7$).

Table 9: Sensitivity Analysis: II

Model 1: $\phi=0.3$; $\beta=0.4$.					Model 2: $\phi=0.5$; $\beta=0.6$.				
	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)	
s_L	0.6	0.65	0.7	0.75	0.6	0.65	0.7	0.75	
θ	0.036	0.0552	0.088	0.1725	0.0477	0.0693	0.1063	0.2025	
r	0.0504	0.0508	0.0502	0.0503	0.0504	0.0509	0.0497	0.0508	
y/k_1	0.3111	0.3136	0.3099	0.3105	0.3111	0.3142	0.3068	0.3136	
g_y	0.0152	0.0154	0.0151	0.0151	0.0152	0.0154	0.0149	0.0154	
ψ	2.0094	1.7914	1.6505	1.4187	1.3795	1.2924	1.2475	1.0951	
χ	0.1849	0.197	0.2094	0.2297	0.1848	0.1974	0.2073	0.2321	
u_E	0.1295	0.152	0.1787	0.2162	0.1226	0.1334	0.1676	0.1976	
u_I	0.1056	0.0897	0.0702	0.048	0.1064	0.0918	0.0711	0.0495	
Model 3: $\phi=0.6$; $\beta=0.7$.					Model 4: $\phi=0.7$; $\beta=0.8$.				
	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)	
s_L	0.6	0.65	0.7	0.75	0.6	0.65	0.7	0.75	
θ	0.055	0.0722	0.1125	0.207	0.0522	0.0722	0.1125	0.2025	
r	0.05	0.0502	0.0507	0.0509	0.0504	0.0492	0.0498	0.05	
y/k_1	0.3086	0.3099	0.3130	0.3142	0.3111	0.3037	0.3074	0.3086	
g_y	0.015	0.0151	0.0154	0.0154	0.0152	0.0146	0.0149	0.015	
ψ	1.1881	1.2035	1.1172	1.0318	1.1715	1.1644	1.0906	1.0191	
χ	0.1835	0.1949	0.2113	0.2323	0.1849	0.1911	0.2076	0.2285	
u_E	0.1841	0.1394	0.1478	0.1928	0.1172	0.1346	0.1526	0.1838	
u_I	0.0984	0.0903	0.0654	0.0498	0.1071	0.0894	0.069	0.0498	

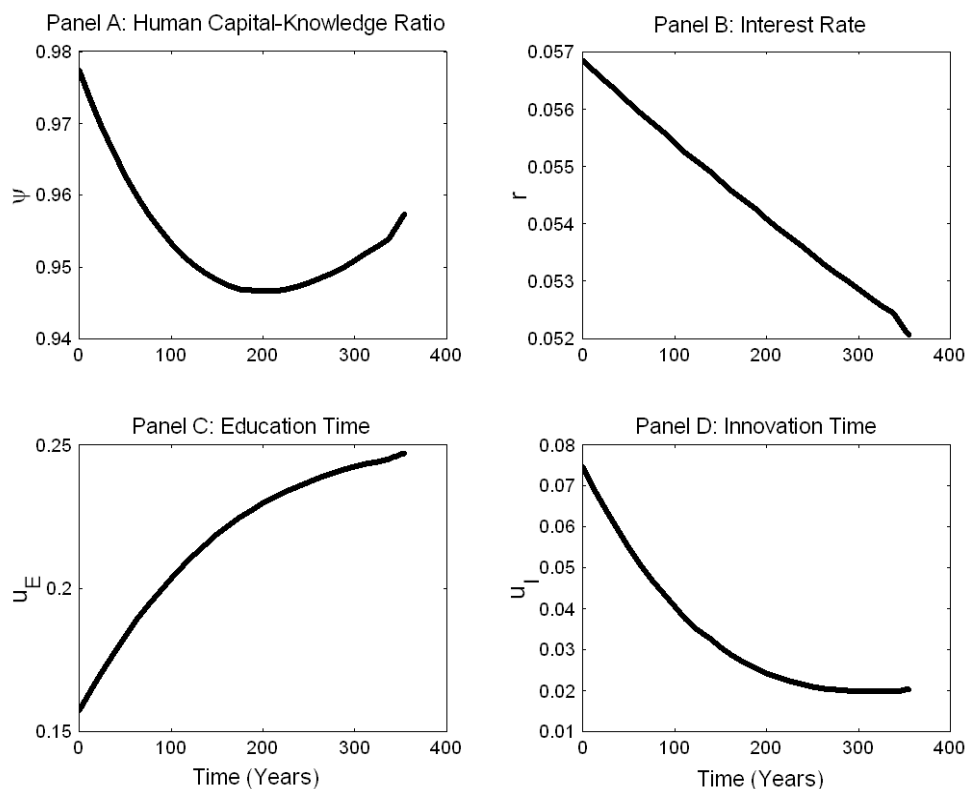
Note. – Sensitivity analysis associated with Table 5.

Figure 1: Two-dimensional path in an education-innovation growth model



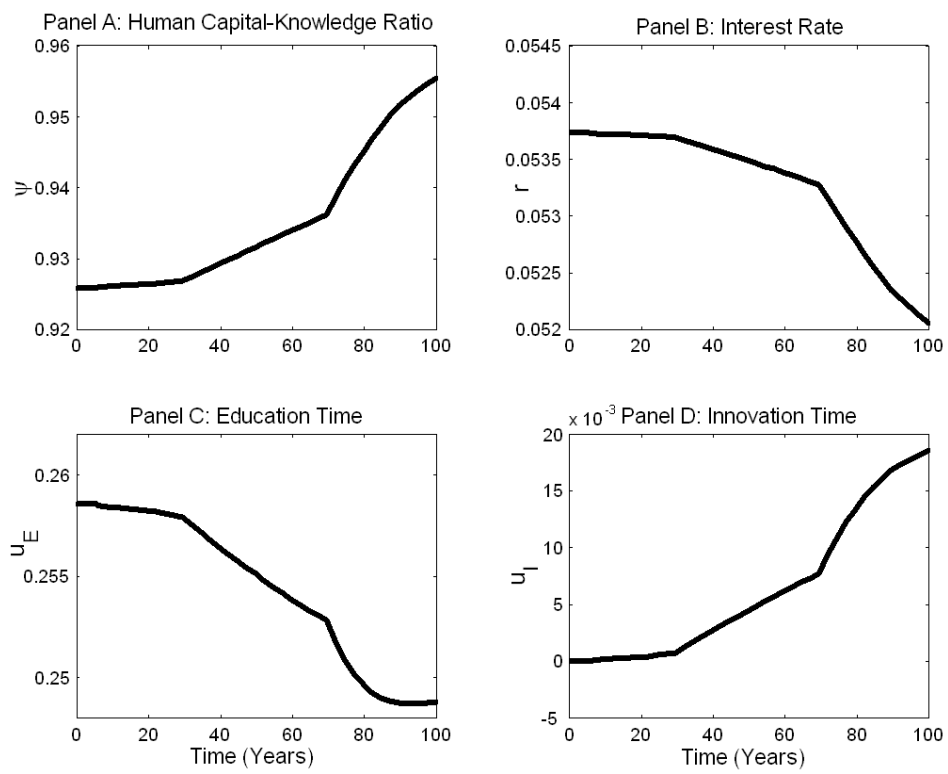
Note. – Any trajectory starting from any of the four regions converge to the node (r^*, ψ^*) . Point A and D lie along the direction of the eigenvector v_{21} (see section (4)).

Figure 2: Productivity shock and destruction of physical capital



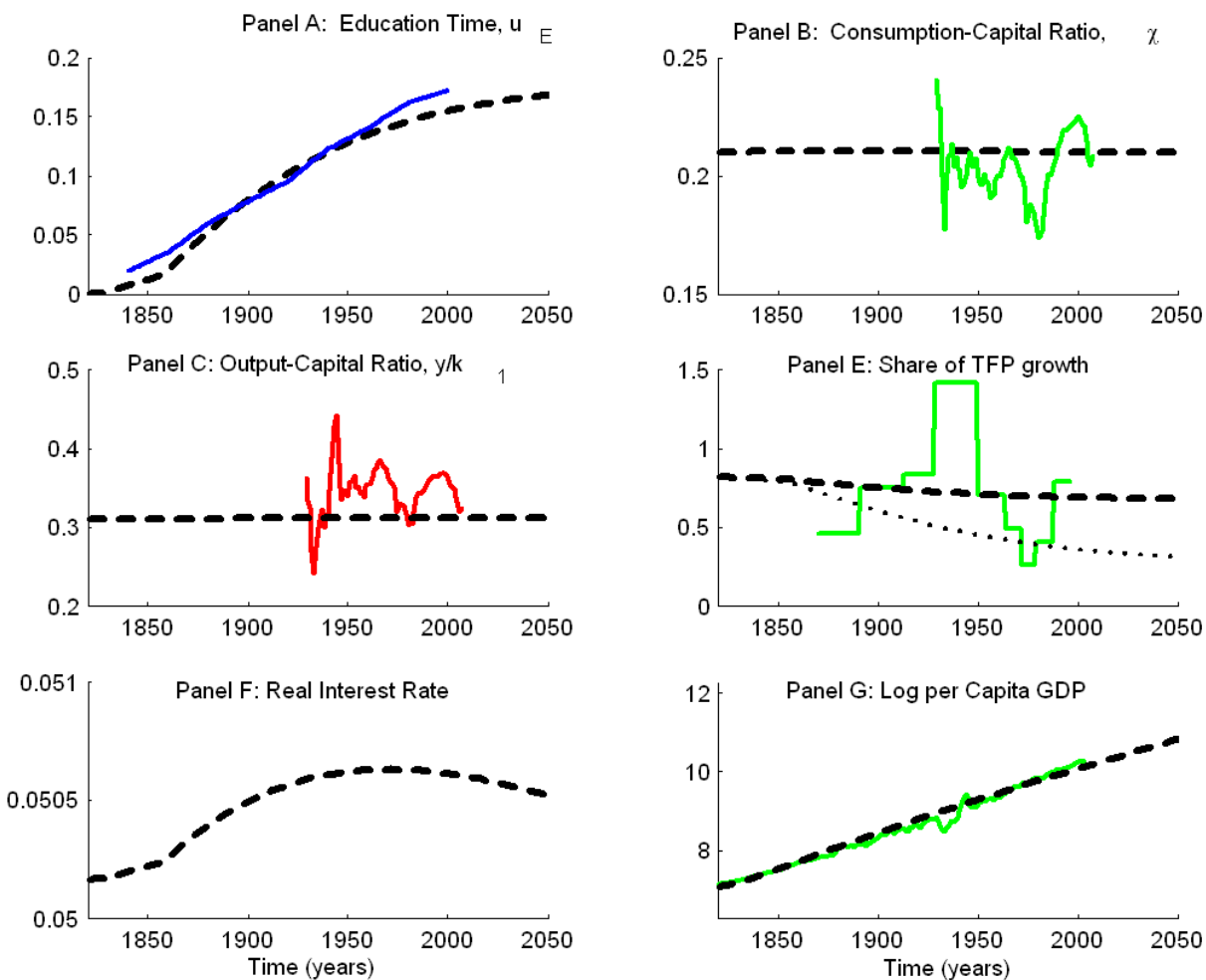
Note. – Adjustment process when the economy is hit by a positive productivity shock (6.9%) or by a negative shock on physical capital (11.4%). The underlying parameters are shown in row (2) of Table (3). The immediate effect of either shock is a sudden increase of the interest rate, which goes from 5 to 5.7%, and of production time, which rises 6.4%.

Figure 3: Inflow of knowledge and destruction of human capital



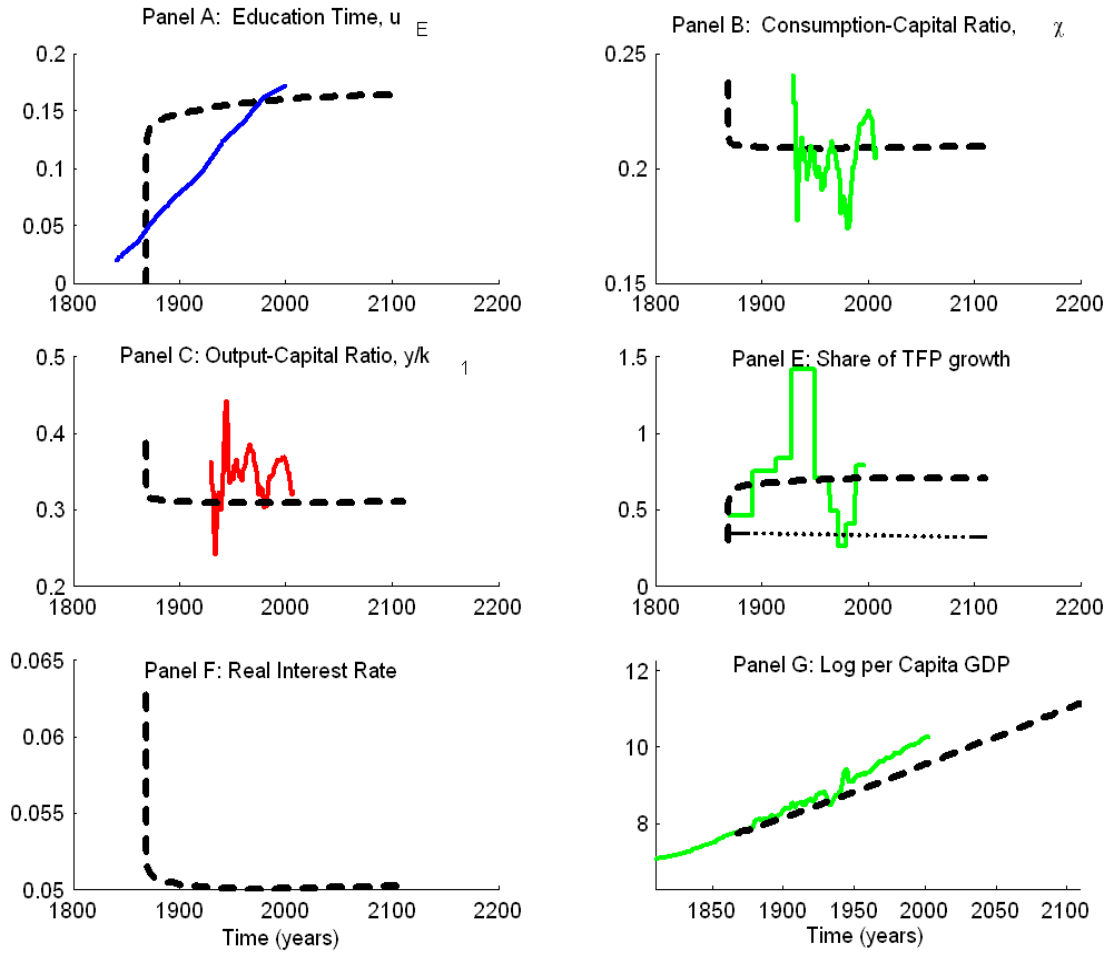
Note. – The shock to n is 5.45%. This brings u_I temporarily to zero, causes the interest rate to jump to 5.38% and u_E to 0.259.

Figure 4: Calibration of the Transitional Dynamics



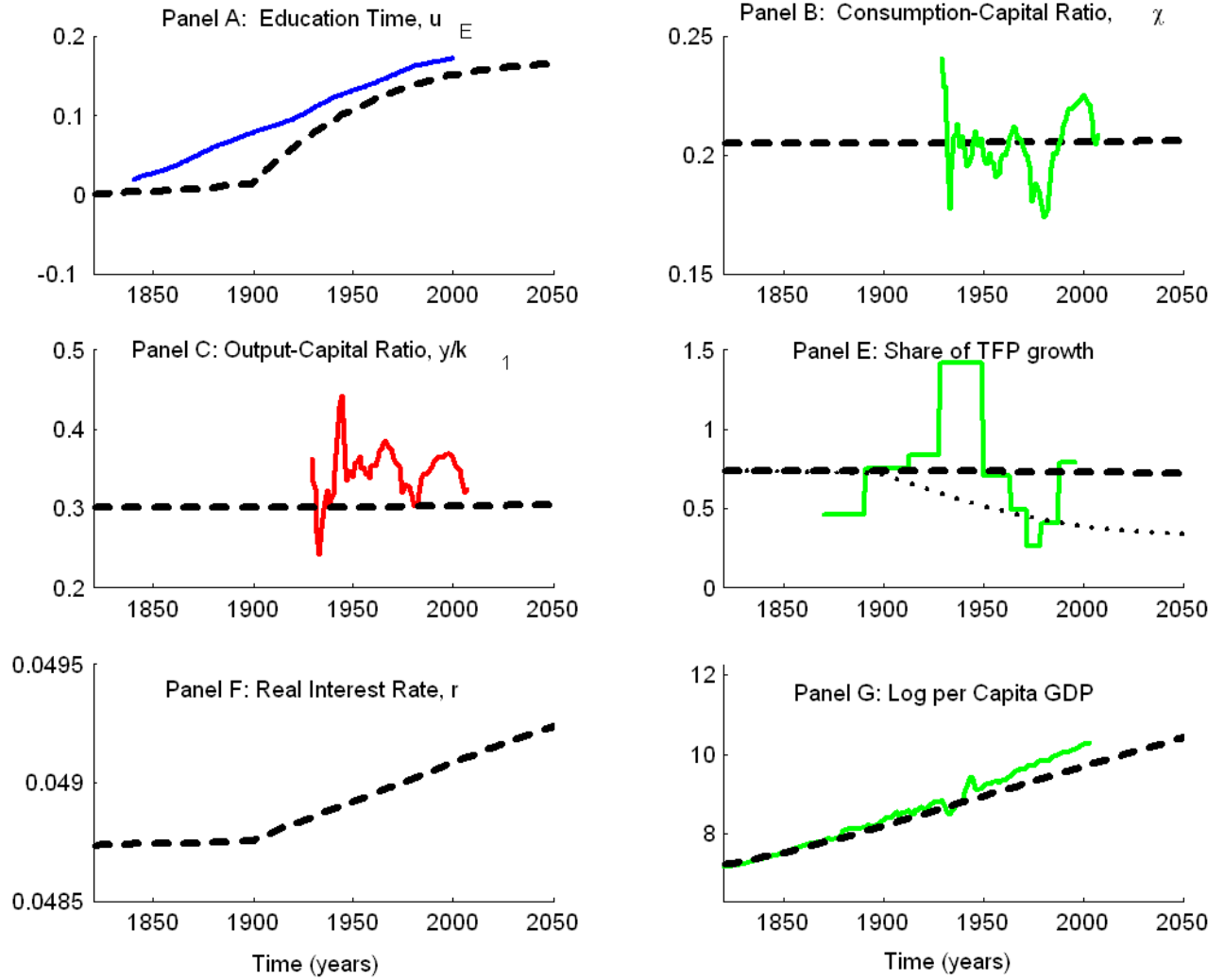
Note. – The dashed line plotted in Panels A through F represent the trajectories implied by the model under the baseline parameters in row (1), Table (3). The continuous lines show the relevant U.S. time series. Details on how u_E was estimated are in Table (5). For a description the actual TFP ratio (continuous line) see note of Table (6). Panel C plots two dashed lines. The top one includes the contribution of both technological progress and education, whereas the bottom one includes innovation only.

Figure 5: Sensitivity Analysis of the Transitional Dynamics (1)



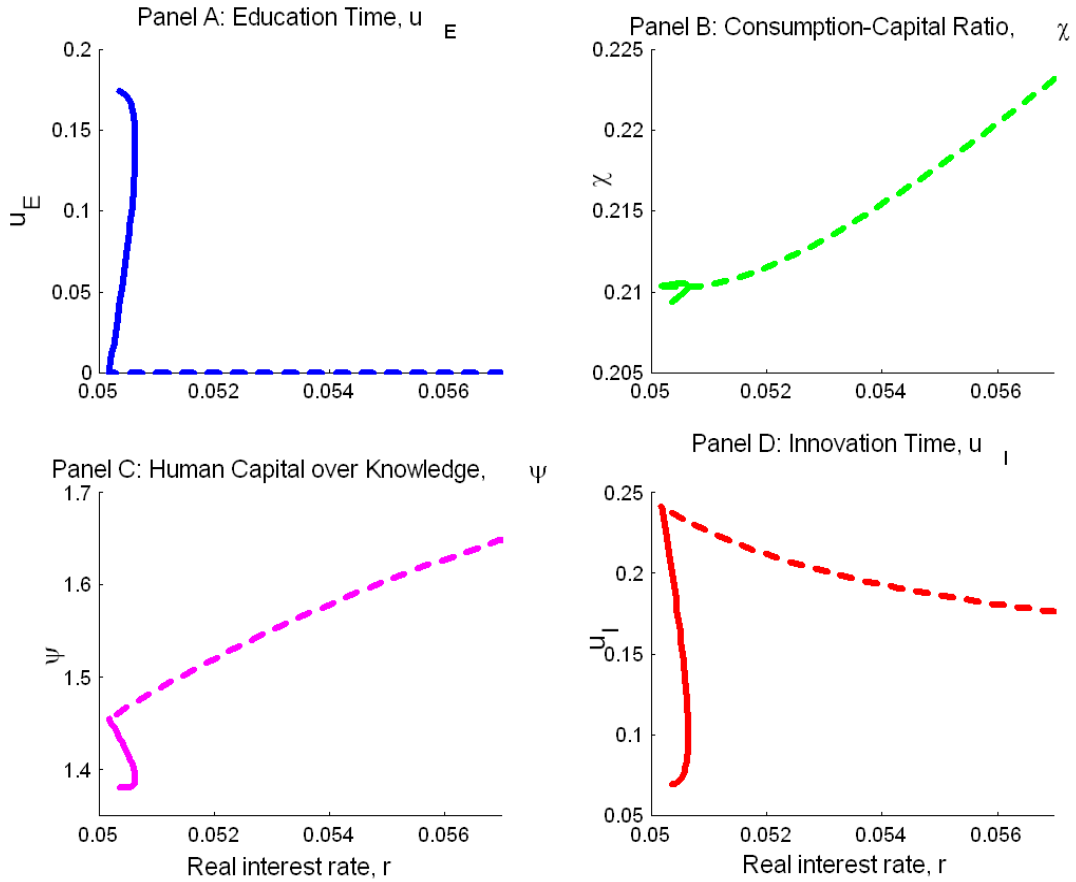
Note. – The underlying parameters are the same as those used in Fig. (4), except that $\beta = 0.9$ and $\theta = 0.115$.

Figure 6: Sensitivity Analysis of the Transitional Dynamics (2)



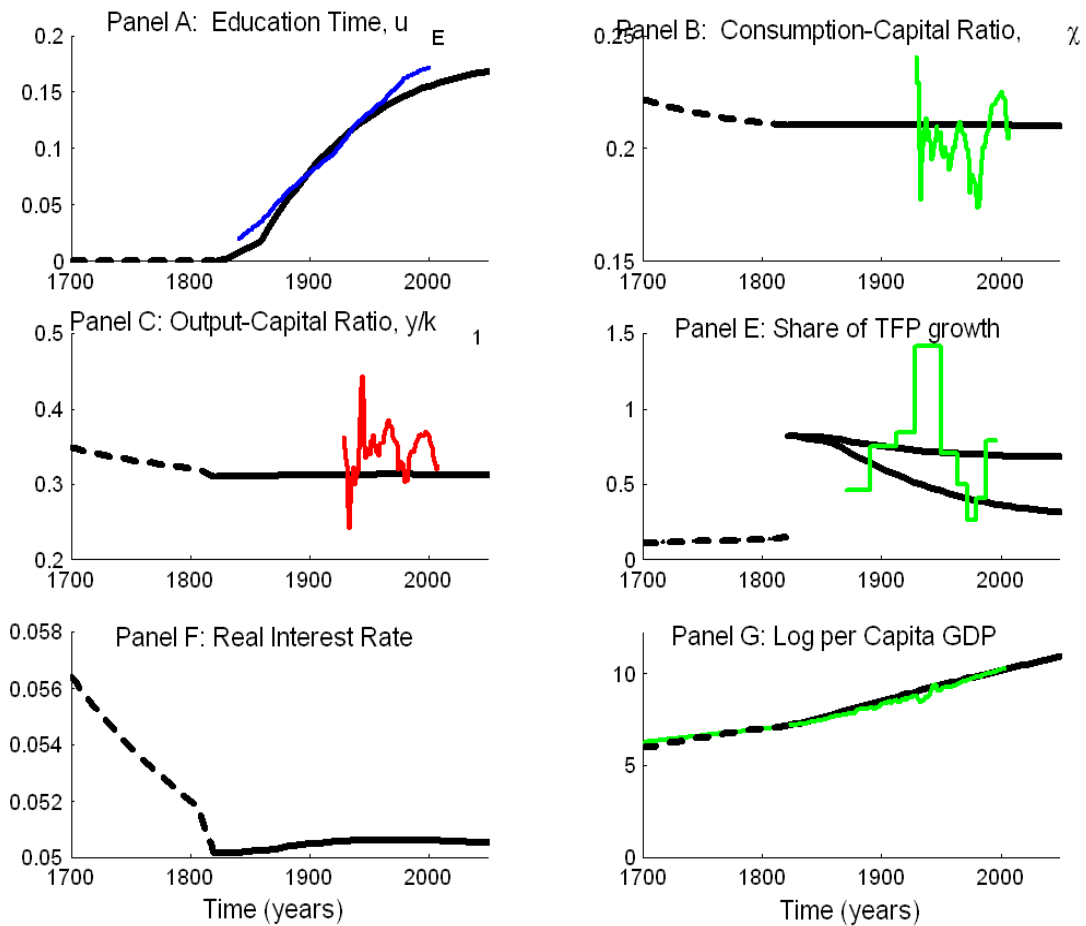
Note. – The underlying parameters are the same as those used in Fig. (4), except that $\beta = 0.6$, $\phi = 0.5$, and $\theta = 0.1063$.

Figure 7: Two-dimensional transition paths over two stages of development



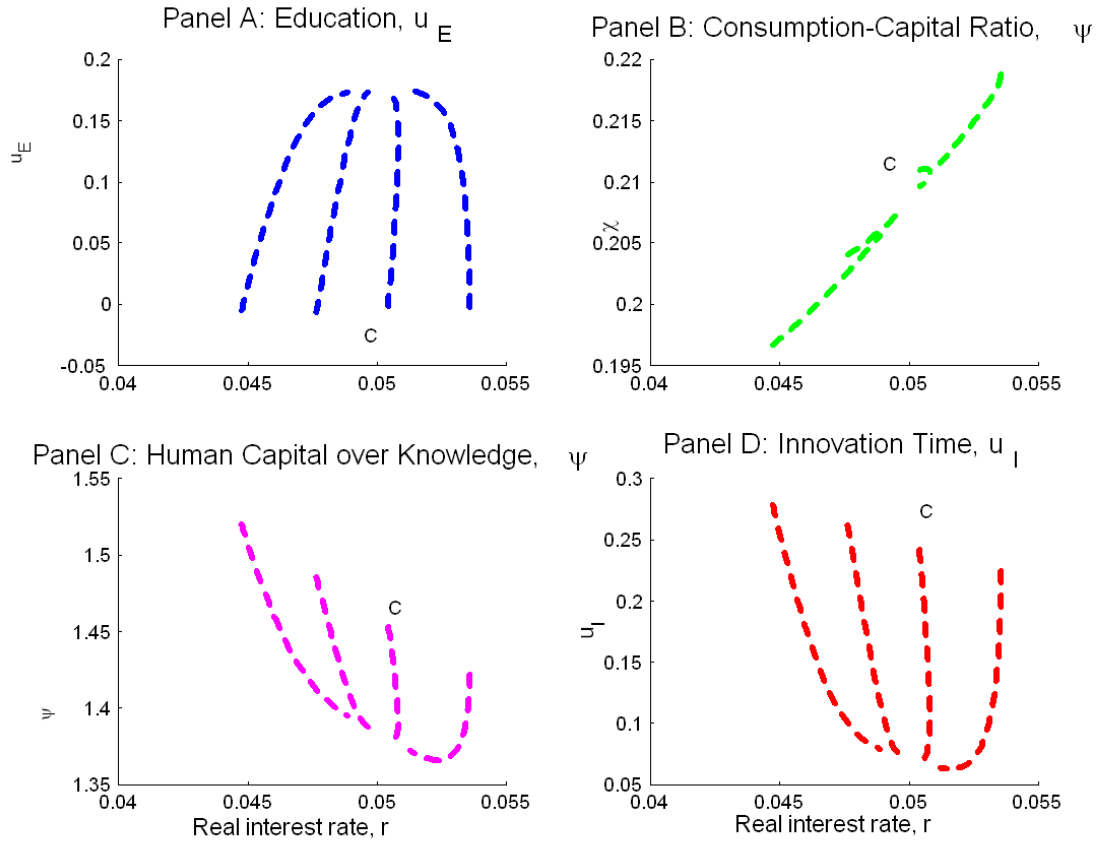
Note. – The dashed lines refer to the innovation-only economy, whereas the continuous line refer to the innovation-education economy. The simulation is based the same baseline parameters (Table 3), row (1)) for both regimes, except that $\theta = 0.058$ in the innovation-only economy to account for the slower income growth of the pre-education development stage.

Figure 8: Time-trajectories over two stages of development



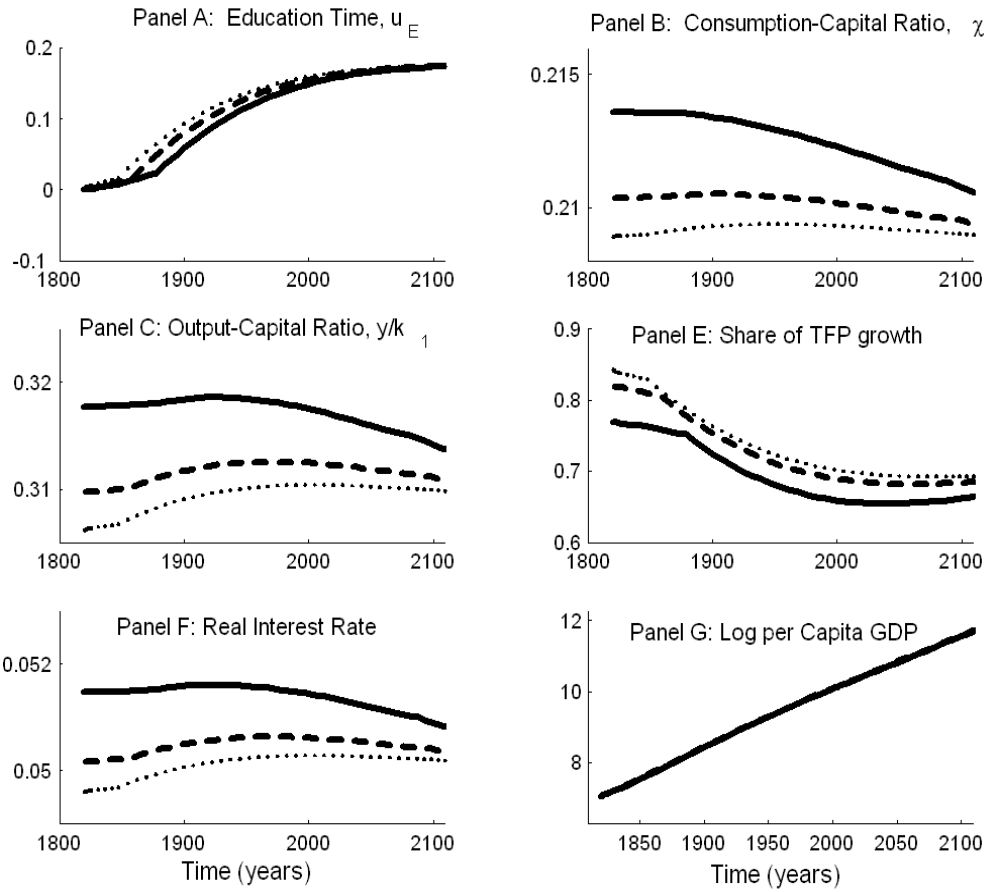
Note. – The plots are the time trajectories associated with the transition paths described in Fig (7). The jump observed in Panel (D) is due to the variation of θ .

Figure 9: Difference in initial conditions and convergence: Phase diagram



Note. – The plots show the convergence of four economies that share the same underlying parameters (table (3)) but have different the initial conditions. The trajectories indicated with 'C' refer to economy calibrated in section (5). The four economies converge to the same steady state.

Figure 10: Convergence to the steady state



Note. – The graphs plot the time trajectories of the economies represented in Fig. (9), other than 'C'.