BIASED TECHNOLOGICAL CHANGE, IMPATIENCE AND WELFARE

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January 2009[‡]

Abstract

In this paper we use an OLG model where agents are heterogeneous within each generation, differing in their impatience rate. We show that the effects of a capital-using technological change are not symmetric between agents and can cause a reduction in consumption. The asymmetry in impatience rates has consequences on the benefits derived from technological change for further generations. Lower impatience rates lead to higher capital levels, and to higher levels of consumption provided that the economy has enough capital per capita.

Resumen

En este artículo utilizamos un modelo de generaciones traslapadas con heterogeneidad en la tasa de impaciencia para mostrar que los efectos de un cambio tecnológico aumentador de capital no son simétricos en los agentes y pueden conllevar una reducción en el consumo. La asimetría en la tasa de impaciencia de los agentes en un período, tiene consecuencias sobre los beneficios del cambio tecnológico para las generaciones futuras.

Keywords Biased Technological Change, Social Welfare, Overlapping Generations

JEL Classification O33, O40, I31

[†]This article won the first prize, category A, at the VI Forum of Students of Economícs, Finance and International Trade at Universidad del Rosario, Bogotá, Colombia, and at the "Yachaywasi" Economics Students Writing Contest at Universidad de Antioquia, Medellín, Colombia. It was also presented at the forum "Theory and Evidence of Growth, Trade and Economic Development, with Special Reference to Latin America" at Instituto Politécnico Nacional, Ciudad de México, México.

[‡]We are grateful to Hernando Zuleta, Oscar Ávila, Enrique Gilles and the students from the Biased Innovations course at Universidad del Rosario for comments on earlier versions of this paper. We also thank the members of the research group "Los Apóstoles" and the students and teachers from Universidad del Rosario, Universidad de Antioquia and Instituto Politécnico Nacional. Any remaining error is our own responsability.

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Contents

1 Introduction

Recently, biased innovation models have gained acceptance.[1] This literature makes extensive use of homogeneous agents models. However, the effects of this type of innovations on different individuals can vary substantially. Indeed, in an overlapping generations framework this type of technological change may reduce the income of young people (see Bertola (1996); and Zuleta and Alberico (2007)). These OLG models, however, only explore the consequence of one source of heterogeneity. On top of that, Bertola (1993) and Bertola (1996) show that change in factor shares can have different effects on different types of agents and non monotonous effects on welfare. We contribute to the literature including heterogenous preferences, in particular, heterogeneous discount factors.

We use a two period overlapping generations model where agents are heterogeneous within each generation, differing on their impatience (or discount) rate. While the heterogeneity might also be due to differences in endowments or utility functions, for the sake of simplicity, we focus only on one source of heterogeneity.. Biased innovations are modeled just as an exogenous increase

[1]Zeira (1998), Acemoglu (2002), Boldrin and Levive (2002), Peretto and Seater (2007) and Zuleta (2008b), among others, provide models of this type.

in the share of capital in the production function, so innovations are of the factor-saving type. As far as we know, our model is the first one involving factor saving innovations and heterogeneous agents within each generation.

In our model, although impatience rates do not change the qualitative effects of technological changes on welfare, they play a role in determining the magnitude of consumption and welfare changes, which are asymmetric among agents. The heterogeneity in discount rates might also determine the rate growth of capital, thus determining the effect of technological change for successive generations.

Kennedy (1964) and Kennedy (1973) introduce the models of biased innovations, arguing that firms change their production technology in order to reduce their costs. Therefore, factor saving innovations will be preferred if capital is more abundant and has a smaller price. However, in these pioneer models, consumers do not play an active role. Recently, some scholars have revisited this topic using general equilibrium framework. Zeira (1998) explains that non neutral technological change can explain permanent income differences among countries. Acemoglu (2002) shows how the effect of biased technological change depends on the elasticity of substitution between inputs and explains how the effects of innovations change as the abundance and relative intensity of factors varies. Peretto and Seater (2007) and Zuleta (2008b) develop endogenous growth models with labor saving (or eliminating) technological progress and show that the economy might stagnate with zero growth or grow perpetually as in the AK model. These models provide a theory of endogenous industrialization.[2]

All these are continuous time models, where consumers are homogenous. In contrast, ours is a discrete time model with heterogeneous agents.

Zuleta (2004) and Zuleta and Alberico (2007) develop an overlapping generations model with factor saving innovations showing that the effects of technological change depend on the initial conditions of the economy and that the relation between income distribution and technological change may be complex. In these models, however, agents are homogeneous within generations. Additionally, these authors do not realize welfare analysis.

The rest of the paper is organized as follows: section 2 shows the theoretical model. Section 3 shows numerical results. Section 4 concludes and discusses possible extensions.

[2]One standard result in this literature is that factor shares should be positively correlated to the relative abundance of reproducible factors and, consistently, to percapita income evels. The empirical evidence seems to support this result (Caselli and Feyrer (2007) , Zuleta (2008a) and Krueger (1999))

2 A model with heterogeneity in impatience rates

2.1 Framework

We use a standard two-period overlapping generations model. There is a continuum of agents in this economy: they are indexed by i and distributed over the $(0,1)$ interval. They are differentiated by their impatience rate β^i. The utility function of each individual is given by:

$$U^i = \ln c_t^i + \beta^i \ln c_{t+1}^i \tag{1}$$

Where c_t^i stands for consumption of the ith individual on period t. There are two inputs, labor L and capital K. While capital can be accumulated, labor is a non reproducible factor. All agents have the same labor endowment, and every agent is able to save, accumulating capital for the second period Labor income is distributed between consumption and savings $(2a)$ on the first period, the last of these defining the capital stock for each agent in the next period $(2b)$. Consumption in the second period depends of this stock $(2c)$. These relationships are summarized in the following equations:

$$w_t = c_t^i + s_t^i \tag{2a}$$
$$s_t^i = K_{t+1}^i \tag{2b}$$
$$c_{t+1}^i = (1 + r_{t+1})s_t^i \tag{2c}$$

Where r is the interest rate. There's a representative firm that produces an unique consumption good with a Cobb-Douglas production function:

$$Y_t = AK_t^\alpha L_t^{1-\alpha} \tag{3}$$

Since all agents have the same labor endowment, and are indexed over $(0,1)$, labor supply is fixed and equal to 1, so we can define $k = \frac{K}{L} = K$.

Given this production function, if factor markets are competitive, and setting the final good as the *numeráire*, factor prices are given by:

$$w_t = (1-\alpha)AK_t^\alpha \tag{4}$$
$$r_t = \alpha AK_t^{\alpha-1} \tag{5}$$

Each consumer's problem is

$$\underset{C_t^i, C_{t+1}^i}{Max} \ \ln C_t^i + \beta^i \ln C_{t+1}^i \quad s.t \quad w_t = c_t^i + \frac{c_{t+1}^i}{1 + r_{t+1}} \tag{6}$$

Solving this problem, we find the usual consumption ratio that arises from the canonical overlapping generations model, as in Diamond (1965):

$$\frac{c_{t+1}^i}{c_t^i} = \beta^i (1 + r_{t+1}) \tag{7}$$

Consumption and savings for each individual in each period are given by:

$$c_t^i = \frac{1}{1+\beta^i} w_t = \frac{(1-\alpha) A K_t^\alpha}{1+\beta^i} \tag{8}$$

$$c_{t+1}^i = \frac{\beta^i (1 + r_{t+1}) w_t}{1+\beta^i} = \frac{\beta^i}{1+\beta^i} (1-\alpha) A K_t^\alpha (1 + \alpha A K_{t+1}^{\alpha-1}) \tag{9}$$

$$s_t^i = K_{t+1}^i = \frac{\beta^i}{1+\beta^i} (1-\alpha) A K_t^\alpha \tag{10}$$

The economy's total saving in the first period is given by:

$$S_t = K_{t+1} = \int_0^1 (1-\alpha) A K_t^\alpha \left(\frac{\beta^i}{1+\beta^i} \right) di \tag{11}$$

and, since we have assumed $L = 1$, we can rewrite (9) as:

$$c_{t+1}^i = \frac{\beta^i}{1+\beta^i} (1-\alpha) A K_t^\alpha \left\{ 1 + \alpha A \left((1-\alpha) A K_t^\alpha \int_0^1 \left(\frac{\beta^i}{1+\beta^i} \right) di \right)^{\alpha-1} \right\} \tag{12}$$

2.2 Equilibrium and Steady state

An equilibrium in this economy is a sequence of aggregate capital stock, agent consumption and factor prices $\left\{ K_t, (c_t^i)_{i\in[0,1]}, r_t, w_t \right\}_{t=0}^{\infty}$ such that the factor price sequence is given by (5) and (4), consumption is given by (8) and (9) and capital evolves according to (11). The steady state is defined in the usual way: setting $K_t = K_{t+1}$, the steady state levels of capital and consumption are given by

$$K_{ss} = [A (1-\alpha) G]^{\frac{1}{1-\alpha}} \tag{13}$$

Existence of the steady state in the overlapping generations economy under this model's assumptions is warranted, see Barro and Sala-i Martin (2004) . Where $G = \int_0^1 \left(\frac{\beta^i}{1+\beta^i} \right) di$. Notice that the expression tends to zero as α goes to one. This is not surprising, meaning that in our model, biased technological change is unable to generate long run economic growth, unlike neutral technological change. Zuleta (2004) shows that in an overlapping generations model with bequests, steady state levels of consumption and savings are greater than cero when $\alpha = 1$. Replacing (13) in (8) and (9) yields expressions for steady state levels of first and second period consumption. Writing c_1 as first period consumption and c_2 as second period consumption, the steady state levels are given by

$$c_{1,ss}^{i} = \frac{[(1-\alpha)A]^{\frac{1}{1-\alpha}}\, G^{\frac{\alpha}{1-\alpha}}}{1+\beta^{i}}$$

$$c_{2,ss}^{i} = \frac{\beta^{i}}{1+\beta^{i}}\,[(1-\alpha)A]^{\frac{1}{1-\alpha}}\, G^{\frac{\alpha}{1-\alpha}}(1+\frac{\alpha}{(1-\alpha)\, G})$$

Notice that smaller impatience rates lead to higher steady state consumption levels.

2.3 Effects of exogenous technological change

We now turn to examine the effects of a capital-using exogenous technological change in this economy. Bertola (1996) shows that, in a continuous time overlapping generations model, higher labor income shares might lead to either larger or smaller economic growth, depending on the intertemporal elasticity of substitution and under certain conditions over the parameters of the model. Our objective is to analyze not only the effect of income shares on economic growth, but the effect on each individual's welfare depending on his discount rate.

Capital-using biased technological change is seen as technological change leading to higher relative use of capital in the production process. In this case, we can see technological change as an increase in α.

As shown in (1) each individuals utility depends on consumption on each period. Overlapping generations models assume individuals choose their consumption and savings levels, c_t^i and s_t^i, based on their wage w_t and the expected interest earnings r_{t+1} on accumulated capital. These, in turn, define consumption on the second period $c_{t+1}^i = (1 + r_{t+1})\, s_t^i$. So each individuals' welfare depends on the impact of technological change over consumption decisions, i.e. changes in equilibrium levels of c_t and c_{t+1} when α changes. These changes will depend on two facts: changes in wages and interest rates produced by the change in α (14) and also, whether technological change is predicted by agents. If technological change occurs after consumption decisions have been taken (an unexpected technological change), the impact on welfare will be different to the one produced when technological change occurs before consumption decisions have been taken (an expected technological change).

$$c_t^i = \frac{1}{1+\beta^i} w_t \;\rightarrow\; \frac{\partial c_t^i}{\partial \alpha} = \frac{1}{1+\beta^i}\frac{\partial w_t}{\partial \alpha} \tag{14}$$

$$c_{t+1}^i = \frac{\beta^i(1+r_{t+1})w_t}{1+\beta^i} \;\rightarrow \frac{\partial c_{t+1}^i}{\partial \alpha} = \frac{\beta^i}{1+\beta^i}\left(\frac{\partial r_{t+1}}{\partial \alpha} w_t + (1+r_{t+1})\frac{\partial w_t}{\partial \alpha}\right).$$

2.3.1 Unexpected Technological Change

Let us assume the change in α happens between periods t and $t+1$. The effect of technological change will be seen from period $t+1$ onwards. Since the change is not predicted, none of the agents will be able to change his consumption decisions optimally. Consumption in the first period remains the same, since the wage w_t remains unaltered. However, second period consumption changes as the interest rate r_{t+1} changes. The agent's welfare changes, and only increases if the interest rate does.

Differentiating (5) evaluated at $t+1$ respect to α yields:

$$\frac{\partial r_{t+1}}{\partial \alpha} = A\left\{[(1-\alpha)AK_t^\alpha G]^{\alpha-1} + \alpha \frac{\partial (K_{t+1}^{\alpha-1})}{\partial \alpha}\right\}$$

Using (11), and differentiating $(K_{t+1})^{\alpha-1}$:

$$K_{t+1}^{\alpha-1} = [(1-\alpha)AK_t^\alpha G]^{\alpha-1}$$

$$\frac{\partial (K_{t+1}^{\alpha-1})}{\partial \alpha} = [(1-\alpha)GAK_t^\alpha]^{\alpha-1}\left[(\alpha-1)\ln(K_t) - \ln\left[(1-\alpha)AGK_t^\alpha\right] + 1\right]$$

We obtain:

$$\frac{\partial r_{t+1}}{\partial \alpha} = A[(1-\alpha)GAK_t^\alpha]^{\alpha-1}\left\{1 + \alpha\left[(\alpha-1)\ln(K_t) - \ln\left[(1-\alpha)AGK_t^\alpha\right] + 1\right]\right\} \tag{15}$$

The last expression is greater than cero if:

$$K_t < \frac{e^{\frac{1}{\alpha}+1}}{(1-\alpha)AG} \tag{16}$$

where the right hand side of this inequality takes positive values whenever $\alpha \in (0,1)$, and is a convex function of α. For values of α close to 0 or 1, this expression is larger,and it achieves a minimum at $\frac{-1+\sqrt{5}}{2}$.

Following these facts, if $K_t \in \left(0, \frac{e^{\frac{1}{\alpha}+1}}{(1-\alpha)AG}\right)$ a capital-using innovation produces a increase of second period's consumption for all agents. Since consumption in the first period remains constant, we get the following result

Proposition 1. *If* $K_t \in \left(0, \frac{e^{\frac{1}{\alpha}+1}}{(1-\alpha)AG}\right)$, *an unexpected capital-using innovation increases welfare for all individuals. If it is not the case, then it decreases welfare.*

Proof. This follows from the previous facts and from replacing in the utility function (the upper bar stands for variables that remain fixed):

$$U^i = \ln \overline{c_t^i(\alpha)} + \overline{\beta^i} \ln \left(c_{t+1}^i(\alpha)\right)$$

□

We stress the fact that increases in welfare can happen in both labor abundant and capital abundant economies. Welfare may be decreased only if either the stock of capital, or its share α, are high before the change is made.

Individuals that are born after the second period $t+1$ will also be affected by the change in the accumulable factor's productivity. However they will be able to adjust their consumption, so, for them, the change is an expected one. We analyze it in the next section.

2.3.2 Expected Technological Change

If there is an expected shock, so agents know there will be biased technological change before they take their consumption decisions, welfare will change according to changes in consumption choices. However, in this case the wage w_t is also modified, so consumption levels vary in both periods.

Differentiating(4) evaluated at t respect to α:

$$\frac{\partial w_t}{\partial \alpha} = AK_t^{\alpha}[(1-\alpha)\ln K - 1]$$

This expression is larger than cero, so the wage increases, if

$$K_t > e^{\frac{1}{1-\alpha}} \tag{17}$$

So if an economy has a large enough stock of capital, biased technological change increases consumption in the first period. The effect on second period's consumption depends on changes in wages and interest rates. Differentiating consumption levels yields:

$$\frac{\partial c_t^i}{\partial \alpha} = \frac{AK_t^{\alpha}}{1+\beta^i}\left[(1-\alpha)\ln K_t - 1\right] \tag{18}$$

$$\frac{\partial c_{t+1}^i}{\partial \alpha} = \frac{\beta^i A}{1+\beta^i} K_t^{\alpha}\{[1+\alpha A\left((1-\alpha)ACK^{\alpha}\right)^{\alpha-1}]$$
$$[K^{\alpha}\{\alpha[(1-\alpha)\ln K - 1] + (1-\alpha)\ln[(1-\alpha)\,ACK^{\alpha}]\}]\} \tag{19}$$

From these expressions, the effect on consumption levels is positive if $K_t > e^{\frac{1}{1-\alpha}}$. Thus, we get the following result:

Proposition 2. *If* $K_t > e^{\frac{1}{1-\alpha}}$*, an expected capital-using innovation increases welfare for all individuals. If it is not the case, then it decreases welfare.*

Proof. This follows from the previous facts and from replacing in the utility funcion.

$$U^i = \ln\left(c_t^i(\alpha)\right) + \overline{\beta^i}\ln\left(c_{t+1}^i(\alpha)\right)$$

□

2.4 Asymmetrical effects

The innovation's effect differs among individuals because they have two heterogeneous characteristics: First, they are not born in the same period. Second, each one of them has a different impatience rate. We now examine the differences in effects caused by these different characteristics:

The overlapping generations model assumes there is an infinite set of agents. So if there is technological change at period $t^* + 1$, for those who are born at period t^* the shock will be unexpected, while for those born on period $t^* + 1$ onwards the shock will be expected. The effects on welfare are summarized in the following results

Proposition 3. *If* $K_t \in \left(e^{\frac{1}{1-\alpha}}, \frac{e^{\frac{1}{\alpha}+1}}{(1-\alpha)AG}\right)$ *and* $\frac{e^{\frac{1}{\alpha}+1}}{(1-\alpha)AG} > e^{\frac{1}{1-\alpha}},$ *a capital using innovation that occurs at* t^* *increases welfare for all individuals born at* $[t^*, \infty)$

Proof. Since $K_t < \frac{e^{\frac{1}{\alpha}+1}}{(1-\alpha)AG}$, from proposition 1, welfare increases for individual born at t^*. From proposition 2, since $K_t > e^{(1/(1-a))}$, welfare increases for individuals born from t^* onwards. □

Proposition 4. *If* $K_t > e^{\frac{1}{1-\alpha}}$ *and* $K_t > \frac{e^{\frac{1}{\alpha}+1}}{(1-\alpha)AG}$ *, a capital using innovation increases welfare for individuals born from* $t^* + 1$ *onwards , and decreases welfare for individuals born at* t^**.*

Proof. Since $K_t > e^{(1/(1-a))}$, from proposition 2, welfare increases for individuals born from $t^* + 1$ onwards.
From proposition 1, since $K_t > ((e^{(1/a)+1})/((1-a)AG))$, welfare decreases for individuals born at $t*$. □

Given the logarithmic utility function, the saving rate does not depend on the interest rate. Biased technological change will increase first period's consumption and savings only if it increases both production and wages.

Summarizing, joining the effects on income shares and the effects on the optimal saving and consumption paths, using (16) and (19), it can be seen that when the economy has enough capital then an expected technological change increases both capital and labor returns, making individuals richer. This makes them increase their consumption levels in both periods. For greater discount rates β^i, the increase (or reduction) in consumption and savings will be smaller. An innovation will have positive effects only if the economy has a relatively abundant stock of capital in period t^*.

Without further restrictions over the model's parameters, it is not possible to describe the effect of technological change on the capital income share over the next period. Although a larger amount of capital at period 1 makes it more likely that the capital income share falls in the next period, the relationship between capital income shares over the two periods is not a monotonous one.

To examine asymmetrical effects on the individuals due to heterogeneity in impatience rates, we examine consumption and saving ratios over individuals. For two individuals i, j, from (8) we have:

$$\frac{c_t^i}{c_t^j} = \frac{\dfrac{1}{1+\beta^i}}{\dfrac{1}{1+\beta^j}} \tag{20}$$

from (10) we find:

$$\frac{s_t^i}{s_j^i} = \frac{\dfrac{\beta^i}{1+\beta^i}}{\dfrac{\beta^j}{1+\beta^j}}$$

and from (9) we have:

$$\frac{c_{t+1}^i}{c_{t+1}^j} = \frac{K_{t+1}^i}{K_{t+1}^j} = \frac{\left(\dfrac{\partial s_t^i}{\partial \alpha}\right)}{\left(\dfrac{\partial s_t^j}{\partial \alpha}\right)} = \frac{\dfrac{\beta^i}{1+\beta^i}}{\dfrac{\beta^j}{1+\beta^j}} \tag{21}$$

Changes in consumption and saving levels depend only on capital levels on period t^*. However, each one of the agents is affected by the innovation in a different way. If the economy is relatively capital abundant, so (16) holds, larger discount rates β^i are associated with a smaller increase in consumption in period t^* and larger increases in savings and consumption levels in t^*+1. More impatient individuals, who have a smaller β^i,have smaller increases in savings and second period consumption, although the increase in first period consumption is larger for them.

So when there is biased technological change, different impatience rates only have incidence on the magnitude of changes in consumption for each agent. What determines the sign of this change, is the relative abundance of capital in the period t^* when the innovation occurs. This abundance depends on impatience rates on the previous period t^*-1.

Let us assume that each generation of agents has different impatience rates[3]. Suppose there is an economy with a small amount of capital, so (16) does not hold. If there is capital-using technological change in this economy, it will reduce overall consumption. However, if $\int_o^1 \beta^i_{t^*-1} di$ were large enough compared to $\int_o^1 \beta^i_{t^*} di$, then the stock of capital could be large enough at $t^*$for (16) to hold. In such case, capital-using technological change would increase overall consumption and welfare. From this reasoning, it can be seen that innovation effects for a generation of consumers depend on the previous generation.

3 A numerical example

In this section we propose a numerical example. We simulate capital, consumption and welfare trajectories for three different kinds of agents with three different impatience rates. Gross utility is our welfare measure. We illustrate three different economies, each one with different settings when the innovation occurs.

Each economy is characterized by the parameters and the initial capital level. In each case, we modify the productivity parameter A in the production function, without modifying the initial capital level. For this example, the biased innovation happens at the 50$^{\text{th}}$ period. The parameters used in simulation are summarized in table 1.

Simulation Parameters			
Initial α	0.4	β_i	0.3
Final α	0.5	β_j	0.6
K_0	4	β_h	0.9
Case 1		A	2.5
Case 2		A	5
Case 3		A	8

Table 1: Simulation parameters

- Case 1 (Figure 1):

When the innovation occurs, the capital level is lower than the critical level $\underline{k} = e^{\frac{1}{1-\alpha}}$, so agents' consumption and welfare levels fall. Notice that the effect is bigger if the agent has a lower impatience rate. However, at the moment

[3]This assumption does not bring dynamic inconsistence issues, since we're dealing with a two period model.

of biased innovation the agents' welfare increases; this is due to the fact that second period consumption increases as the interest rate r_{t+1} changes: this increases welfare for individuals born at t^*.

- Case 2 (Figure 2):

Everyone's welfare increases because when the biased innovation occurs the economy has sufficient capital. In this economy the capital level is between $e^{\frac{1}{1-\alpha}}$ and $\frac{e^{\frac{1}{\alpha}+1}}{(1-\alpha)AC}$. In this case, if an agent has a lower impatience rate, his welfare will increase more.

- Case 3 (Figure 3):

In this economy, there is relative abundance of capital, but the capital labor ratio is higher than $\frac{e^{\frac{1}{\alpha}+1}}{(1-\alpha)AC}$. In this case, individuals born right before the innovation lose welfare (because the change is unexpected).

4 Conclusions

According to a two-period overlapping generations model with heterogeneous agents, a change of impatience rates does not create an ambiguous effect of a biased technological change. However, the heterogeneity affects the magnitude of consumption changes.

When the economy is scarce in capital, a capital using biased innovation reduces everyone's welfare, and the magnitude of the change depends on the impatience rate. In an economy with such characteristics the agents stay in a poverty trap. If a change in the impatience rate at any period is considered, such that β^i increases enough to drive $\int_0^1 \left(\frac{\beta^i}{1+\beta^i}\right) di$ up and to increase capital to a level $K > e^{\frac{1}{1-\alpha}}$, agents born in the following periods are favored by technological change and can escape poverty.

Generations preceded by others with lower impatience rates will be more favored by technological change. This fact has policy implications: biased technological change alone is unable to generate long run economic growth if there is not enough capital and if impatience rates are high. If technological change is endogenous, it is unlikely that capital-using biased technological changes happen if there is insufficient capital. However, if technological changes occur exogenously, their effects are not symmetric and might be prejudicial.

There are several ways to extend the analysis: using a non logarithmic utility function, so saving depends on the interest rate, or analyzing differences in steady state levels of variables due to heterogeneity. Also, an economy where technology is decided by votes of heterogeneous agents can be considered.

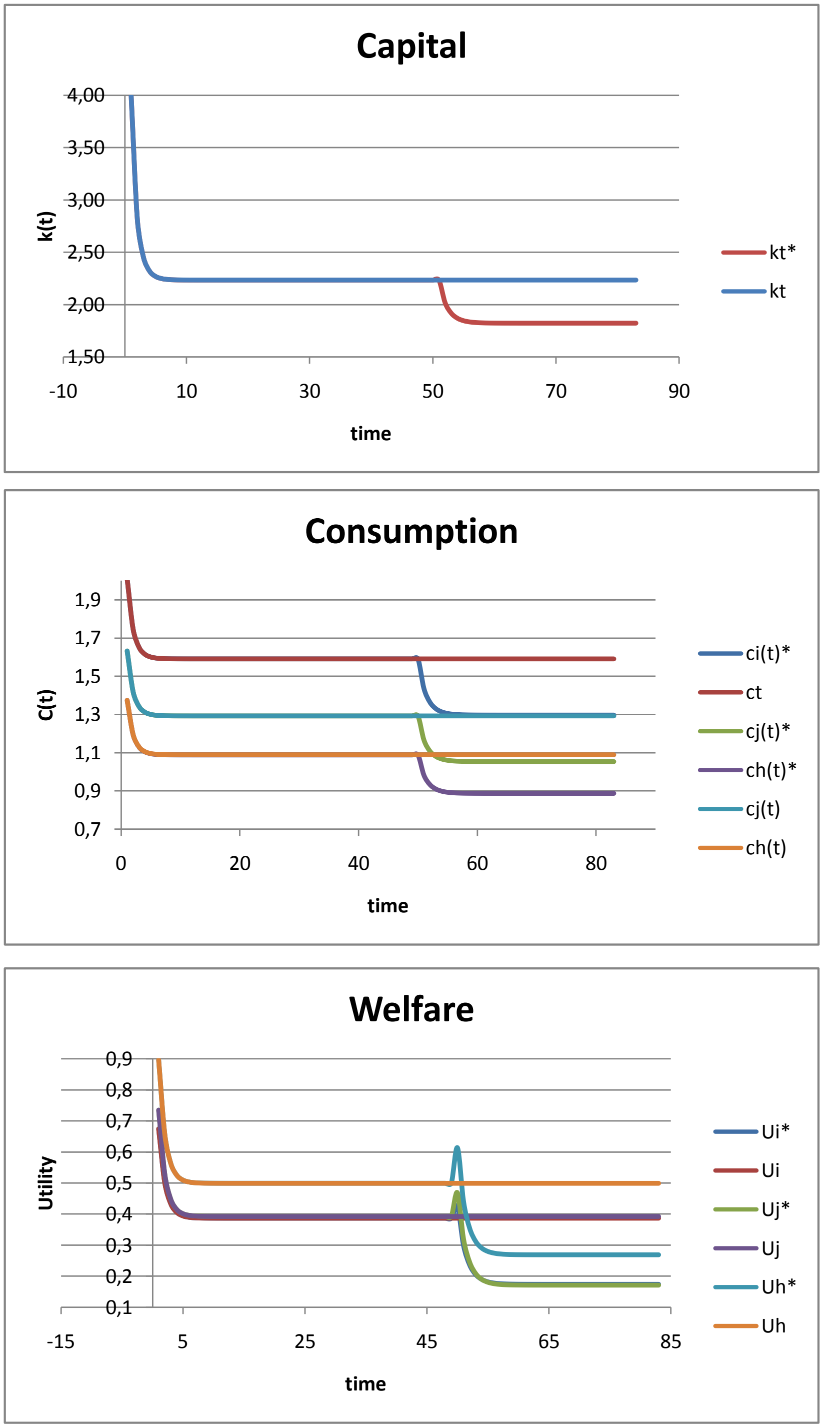

Figure 1: Case 1: Trayectories of variables when $K < e^{\frac{1}{1-\alpha}}$

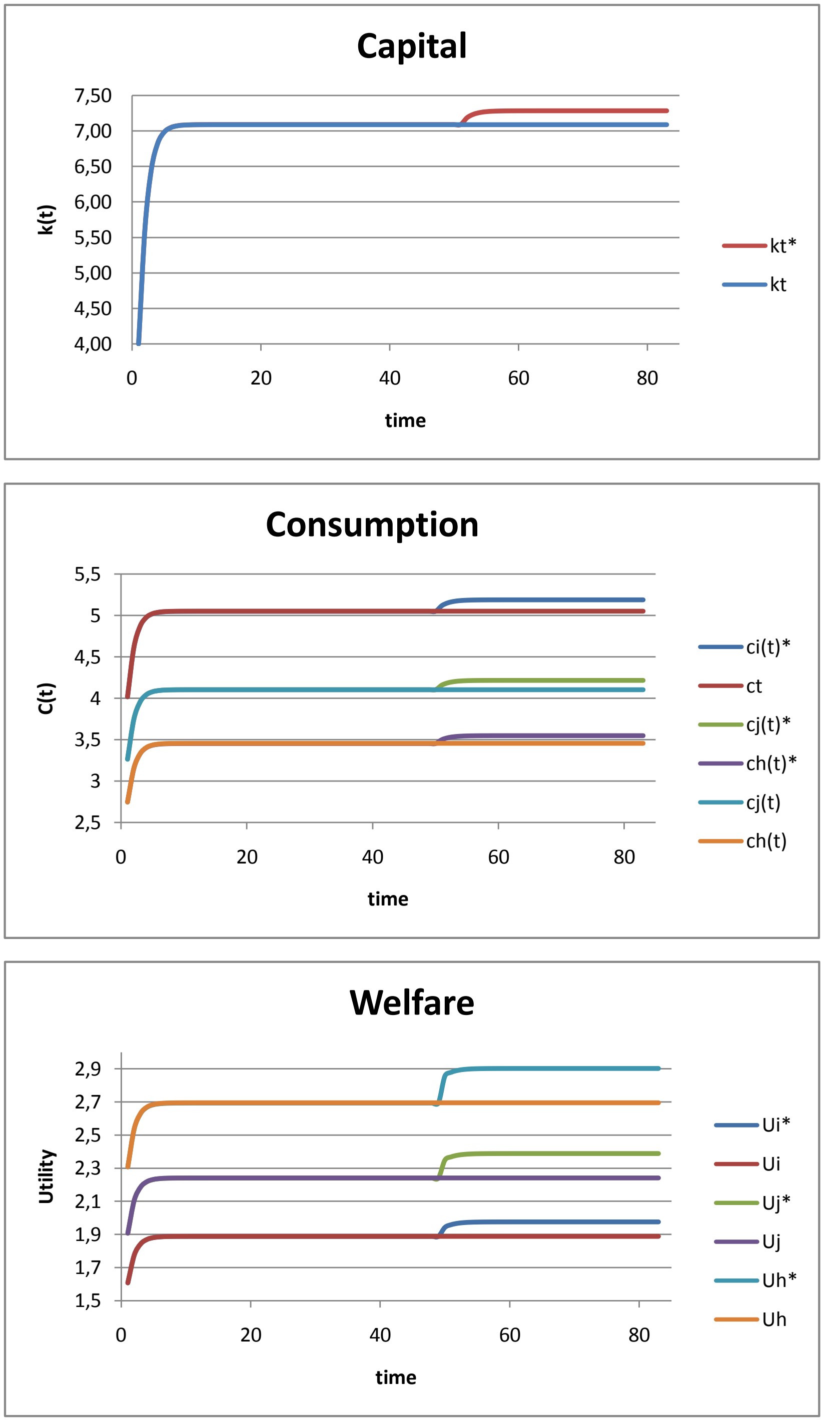

Figure 2: Case 2: Trayectories of variables when $K \in \left(e^{\frac{1}{1-\alpha}}, \frac{e^{\frac{1}{\alpha}+1}}{(1-\alpha)AC}\right)$

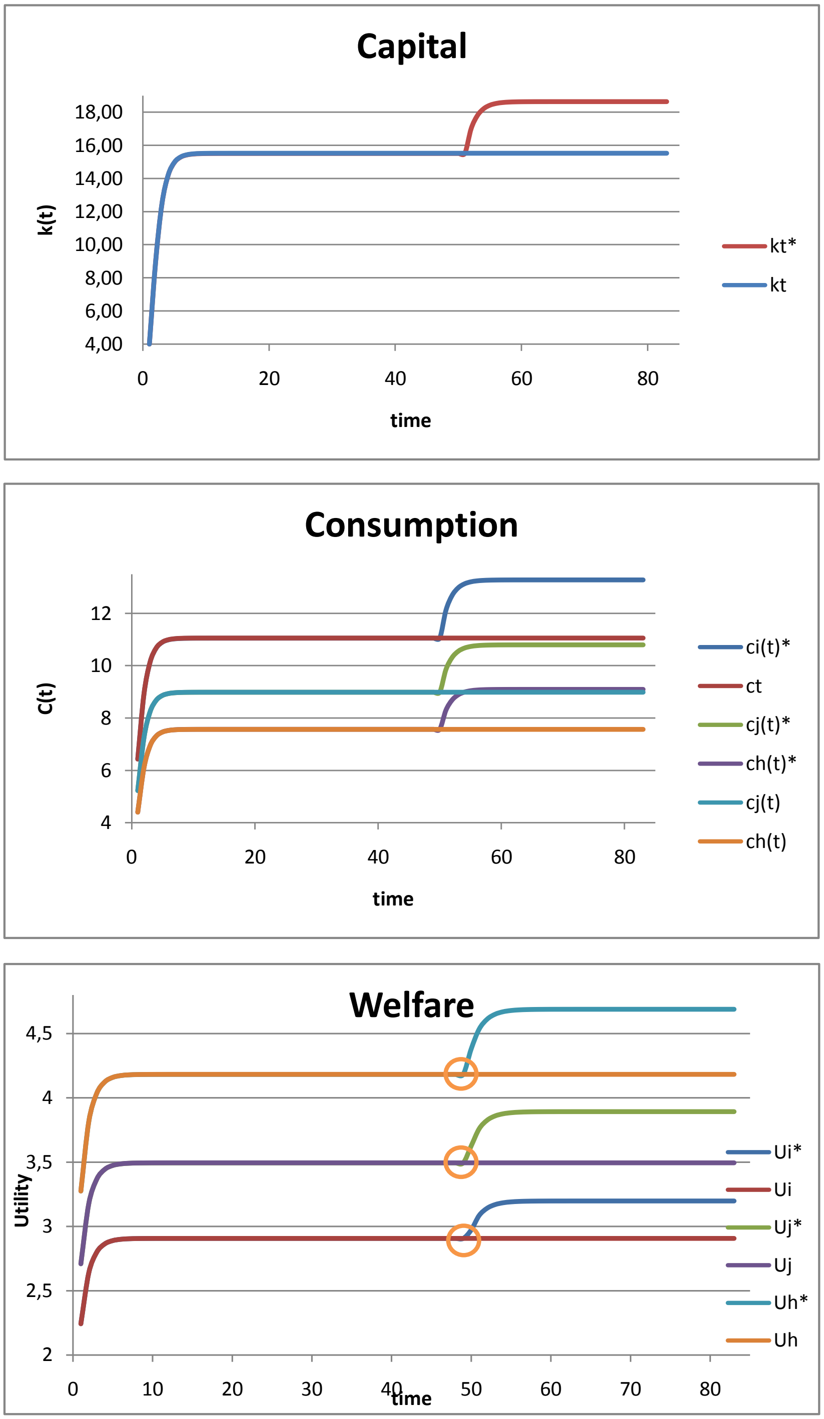

Figure 3: Case 3: Trayectories of variables when $K > \frac{e^{\frac{1}{\alpha}+1}}{(1-\alpha)C}$

References

Acemoglu, D. (2002). Directed technical change. *The Review of Economic Studies 69*(4), 781–809.

Acemoglu, D. (2009). *Introduction to Modern Economic Growth*. Princeton University Press.

Barro, R. and X. Sala-i Martin (2004). *Economic Growth*. The MIT Press.

Bertola, G. (1993). Factor shares and savings in endogenous growth. *The American Economic Review 83*(5), 1184–1198.

Bertola, G. (1996). Factor shares in olg models of growth. *European Economic Review 40*(8), 1541–1560.

Caselli, F. and J. Feyrer (2007). The marginal product of capital. *Quarterly Journal of Economics 122*(2), p535 – 568.

Diamond, P. (1965). National debt in a neoclassical growth model. *American Economic Review 55*(5), 1126–1150.

Kennedy, C. (1964). Induced bias in innovation and the theory of distribution. *The Economic Journal 74*(295), 541–547.

Kennedy, C. (1973). A generalisation of the theory of induced bias in technical progress. *The Economic Journal 83*(329), 48–57.

Krueger, A. B. (1999). Measuring labor's share. *The American Economic Review 89*(2), 45–51.

Peretto, P. F. and J. J. Seater (2007). Factor-eliminating technical change. Working Paper 17, Economic Research Initiatives at Duke (ERID), Available at SSRN: http://ssrn.com/abstract=1270650.

Sala-i Martin, X. (2002). *Apuntes de Crecimiento Económico*. Antoni Bosch Editor.

Zeira, J. (1998). Workers, machines and economic growth. *Quarterly Journal of Economics 113*(4), 1091–1118.

Zuleta, H. (2004). Factor saving innovations and capital income share in olg models. Documentos de trabajo, Instituto tecnológico Autónomo de México.

Zuleta, H. (2008a). An empirical note on factor shares. *Journal of International Trade & Economic Development 17*(3), 379–390.

Zuleta, H. (2008b). Factor saving innovations and factor income shares. *Review of Economic Dynamics 11*(4), 836–851.

Zuleta, H. and S. Alberico (2007). Labor supply, biased technological change and economic growth. *Ensayos sobre Política Económica - Banco de la República 25*(53), 260–286.