On R&D and the undersupply of emerging versus mature technologies

Tom-Reiel Heggedal*
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Statistics Norway, PO 8131 Dep, 0033 Oslo, Norway

Abstract

An important policy question is whether research and development (R&D) in new, emerging technologies should be subsidized more than R&D in other more mature technologies. In this paper I analyze if innovation externalities caused by knowledge spillovers from private firms may warrant a differentiated R&D policy. I find that R&D in emerging and mature technologies should not be subsidized equally. The reason is that R&D in the two technologies is not equally undersupplied in the market due to differences in their knowledge stocks. R&D in the mature technology should be subsidized more when the sum of the output elasticities with respect to labor and knowledge in R&D production is high, while R&D in the emerging technology should be subsidized more when the elasticities are low.

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1 Introduction

A well known result is that the social benefits from research and development (R&D) may be greater than the private benefits from R&D. One of the main reasons is that knowledge generated from private firms' R&D activity spills over to other firms and expands future R&D opportunities (Griliches, 1995; Jones and Williams, 2000; Klette et al., 2000). Consequently, governments should support R&D activity. However, policymakers and environmentalists often claim that R&D in new, emerging technologies requires special attention. One recurring argument is that

emerging technologies need a pull or a push to get started since the social benefits of new knowledge in such technologies are greater than the benefits of new knowledge in mature technologies.

Technology development is a catchword in the climate change debate. In particular the development of low carbon emission technologies like wind and solar power, carbon capture and sequestration, and hydrogen technology for the transport sector receive a lot of attention from politicians. A key policy question is whether such technologies warrant more governmental support than other more mature technologies.

In general economists are not so enthusiastic about using public funds to promote R&D in particular technologies on behalf of R&D in other technologies. The main reason is that there may be market failures in all research markets. Innovation externalities, e.g. knowledge spillovers, give rise to undersupply of R&D in both emerging and mature technologies. Thus, R&D subsidies to correct for the undersupply should be uniform as long as the innovation externalities are similar in all technologies. However, the innovation externalities might be directly dependent on the maturity of the technology. The reason for this dependency is that when the pool of knowledge in a technology is large a new idea might not provide the same spillovers to future R&D as when the pool of knowledge is smaller.

In this paper I study how innovation externalities caused by knowledge spillovers may differ between technologies of different maturity. The maturity of a technology is defined here as the accumulated knowledge in that technology, i.e. the size of the knowledge stock. In particular, I ask if the undersupply of R&D from private firms is the same in an emerging technology as in a mature technology.

I find that the governmental support for R&D in emerging and mature technologies should not be equal. The reason is that R&D in the two technologies is not equally undersupplied on a transition path. Thus the maturity of the technologies matters when policy makers are to give socially efficient subsidies to different R&D industries.

The reason maturity matters for optimal policy is that the production of new ideas depends on the accumulated stock of knowledge from previous R&D production. When the knowledge stock increases the private firms get a productivity gain through improved output of conducting R&D. Further, this productivity gain from new ideas is declining in the size of the knowledge stock, i.e. decreasing returns to new ideas (Jones, 1995 and 1999). Since these spillovers to future R&D are external to private firms, R&D activity should be subsidized. The size of the spillovers depends on how large the productivity gain from new ideas is and how many researchers take advantage of the productivity gain, i.e. the level

of R&D activity in future periods. R&D activity is higher in the mature technology due to lower costs, while the productivity gain from a new idea is higher in the emerging technology. That the spillovers are dependent on the knowledge stocks implies that R&D subsidies to the two technologies should not be equal.

Whether the emerging technology or the mature technology is more undersupplied in the market solution depends on the sum of the output elasticities with respect to labor and knowledge in R&D production, i.e. the elasticity of scale. R&D in the mature technology should be subsidized more when the elasticity of scale is larger than one, while R&D in the emerging should be subsidized more when the elasticity of scale is smaller than one. The reason is that the elasticities determine the growth rates of the knowledge stocks in the two technologies. The labor allocation between the R&D industries follows from the relative size of the knowledge stocks. However, in the competitive equilibrium the labor allocation only depends on the current period knowledge stocks, while the social planner takes into account the knowledge stocks over the whole time period. Hence, it is optimal to deviate from the private labor allocation and increase the allocation to the technology that grows faster. The mature technology grows faster when the elasticity of scale is larger than one, while the emerging technology grows faster when the elasticity of scale is smaller than one. Thus, an immature technology should only receive more governmental support than other technologies for a specific range of values for key output elasticities in the R&D production.

Gerlagh et al. (2007) study maturity of technology and public R&D support. They find that the optimal subsidy to a maturing technology falls over time. However, they find this in a model with one technology sector and thus lack the relative consideration of the undersupply between emerging and mature technologies. Further, their finding comes as a result of inefficiencies in the R&D market related to limited patent lifetime. They find that the subsidy rate should be falling because the value of abatement increases rapidly in the beginning as the carbon tax increases. Hence, the early innovators get a smaller share of the benefits from this increase than late innovators because patent lifetime is limited, not because the technology is less developed in the beginning, i.e. a small initial knowledge stock, which is the case in this paper.

I develop a semi-endogenous growth model in the spirit of Jones (1995) in order to analyze how the relative undersupply of R&D depends on the maturity of the technologies. The model has two R&D industries which deliver patents (ideas) in two different technologies; one emerging technology and one mature technology. The productivity of the R&D industries is increasing in the accumulated knowledge stock in the re-

spective technologies. On the balanced growth path the model gives the standard result on scale effects for semi-endogenous growth models, i.e. subsidies to R&D do not influence long-run growth rates. However, innovation policy affects the growth along a transition path and thus affects the level of income (production) on the balanced growth path (Jones and Williams, 2000). On the transition path towards the balanced growth path, R&D subsidies correct for the undersupply of R&D from private firms. This correction speeds up the process of reaching the balanced growth path and gives level effects to income. Perez-Sebastian (2007) shows that the long-run level effects of innovation policies can be substantial in a semi-endogenous growth model. The main focus in the paper presented here is to study the relative undersupply of R&D in the two types of technology outside the balanced growth path. This relative undersupply arising from the different maturity of technologies is, to my knowledge, not studied much.

How different types of technology can be undersupplied in the market is analyzed by Hart (2008). Rather than looking at public support for technology investment, he implements optimal second-best carbon taxes. These taxes may be higher than the Pigouvian level in order to encourage investment in emissions-saving technology at the expense of general production technology. The reason is that the emissions-saving technology may be relatively more undersupplied than the other technology. However, this result is derived from an increased scarcity of the environmental good through a rising shadow price of emissions rather than the maturity of technology, which is the sole cause of the relative undersupply of technology in this paper.

In another study, Kverndokk and Rosendahl (2007) find that newly adopted technologies should be subsidized more than older technologies. Their technology externalities, however, come from learning effects, as opposed to R&D externalities in this paper. In their model the learning effects are strongest for newly adopted technologies so they have higher spillovers than older technologies. Hence, the optimal subsidies decrease over time as the learning effects diminish.

R&D externalities and subsidies to environmental-friendly technologies are studied in Heggedal and Jacobsen (2008). They find that the R&D subsidies should fall over time since spillovers are larger in early periods due to decreasing returns to new ideas. However, they only study subsidies to R&D in one type of technology, and do not analyze the relative undersupply of R&D in technologies with different maturity.

There are several papers that study innovation in multi-sector R&D models with symmetric equilibria (e.g. Smulders and van de Klundert, 1995 and 1997; Young, 1998; Segerstrom, 2000; Li, 2000 and 2002;

Peretto and Smulders, 2002). However, the symmetric equilibria in the respective types of innovation sectors, variant expansion and/or quality improvement, imply that the papers do not study the consequences of differences in R&D productivity between firms. There are several other papers with multi-sector R&D models that have asymmetric equilibria in the innovation sectors (e.g. Acemoglu, 2002; Smulders and Nooij, 2003; Grimaud and Rouge, 2008). However, these papers do not study the implication that the different maturity of technologies has for the allocation of resources between R&D industries.

Two papers with multi-sector R&D models that specifically take the maturity of technologies into account are Doi and Mino (2005) and Reis and Traca (2008)[1]. Doi and Mino (2005) investigate equilibrium dynamics in a model of endogenous technological change with two R&D industries. They find that the relative size of the knowledge stocks matters for the allocation of resources to production of consumer goods and production of R&D. Reis and Traca (2008) analyze the implication of a leading and a laggard technology for long run growth in a model with quality improvement[2]. They find that intersectoral spillovers may prevent a monopolization of the market by the productivity leader and thus prevent a stagnation of growth. Neither Reis and Traca (2008) nor Doi and Mino (2005) account for decreasing returns to new ideas which is done in this paper. Further, both studies only investigate the market equilibrium and do not explore the connection between spillovers, the maturity technologies and optimal policies.

The paper is organized as follows. Section 2 gives an illustration of the core mechanism in the model, while the model is laid out in Section 3. The relative undersupply of the technologies is analyzed in Section 4. Numerical simulations are presented in Section 5, and Section 6 concludes.

2 Illustration

One reason why R&D in emerging and mature technologies may not be equally undersupplied is that a typical patent production function is concave in the amount of previous patents (ideas). A functional form

[1] Although few papers, to my knowledge, investigate the consequences of different maturity for R&D policy in closed economies, there are several studies of cross-country differences in technological development. Many of these studies focus on the distance to the technology frontier and differences in growth rates and income, see for example Aghion and Howitt (2006) and Madsen (2008).

[2] See for example Aghion et al. (2001) for more references on other leader and laggard quality improvement models.

often used in growth models is

$$\dot{A} = \nu L^{\lambda} A^{\phi} : 0 < \lambda < 1, 0 < \phi < 1, \tag{1}$$

where $\dot{A}$ is the production of new patents, L is labor input, ν is an exogenous technology factor, λ is the output elasticity with respect to labor, ϕ the output elasticity with respect to patents, i.e. the spillover parameter, and A is stock of patents accumulated from previous periods, i.e. the knowledge stock. The spillover parameter reflects the effect of the existing knowledge stock on the production of patents. A spillover parameter below one is supported by both theoretical and empirical findings (Jones, 1995 and 1999; Jones and Williams, 2000; Popp, 2002; Gong et al., 2004). With a parameter below one, the model exhibits weak scale effects and long-run growth rate of patents is dependent on the population growth rate, which are the main characteristics of semi-endogenous growth models (Jones, 2005). The patent productivity is increasing in the knowledge stock in the following way:

$$\frac{\partial \dot{A}}{\partial A} = \nu L^{\lambda} \phi A^{\phi - 1} > 0. \tag{2}$$

This productivity gain is the source of the spillover effect. Patents from previous periods lower the cost of producing new patents. The ultimate reason is that ideas are non-rival goods, in the sense that one entity's use of an idea does not diminish the benefit for other entities' simultaneous use. Patent protection rights may give excludability for products based on new ideas. However, this does not prevent others from using that idea to create new ideas, i.e. "standing on the shoulders of giants". This spillover from the production of a patent to all firms that produce patents in later periods is not accounted for by the individual firms, i.e. it is an externality. The spillovers imply that the firms undersupply patent production to the market, and a policymaker should try to correct for this, e.g. by subsidizing R&D.

From (2) it is clear that patents in a mature technology can be produced with less resources than in an emerging technology, where maturity is defined by the size of A. *Ceteris paribus*, the mature technology will be allocated more resources than the emerging technology both in the private and the social (optimal) equilibrium. The amount of labor input in future periods' patent production influences the spillovers from current period production in the following way:

$$El_L(\frac{\partial \dot{A}}{\partial A}) = \lambda > 0. \tag{3}$$

In a future period with high R&D production, the benefits are greater from an increase in the stock of ideas because it lowers costs for a larger

production set. In other words, the benefits are greater when a new patent spills over to more R&D firms and researchers. I find it convenient to name this effect expressed by λ the *spillover size effect*. This effect means that the spillovers are larger in the mature technology than in the emerging technology, which implies that R&D in the mature technology should be subsidized more.

On the other hand, there is the concavity of the production function which implies that the change in patent productivity is smaller when the knowledge stock is larger:

$$El_A(\frac{\partial \dot{A}}{\partial A}) = (\phi - 1) < 0. \tag{4}$$

The reason is that when a new patent is added to a large set of other patents this does not increase the R&D opportunities in future periods as much as an additional patent when there are few other patents. Thus, a new patent in an emerging technology sector provides a greater productivity increase than a new patent in a mature technology. I find it convenient to name this effect expressed by $\phi - 1$ the *spillover depletion effect,* which implies that R&D in the emerging technology should be subsidized more. The relative undersupply of R&D in the two technologies depends on whether the *spillover size effect* or the *spillover depletion effect* dominates.

An example of technologies where the spillover effects may be present is conventional cars with internal combustion engines and hydrogen cars with fuel cells as the energy conversion system. A lot of research has been carried out on internal combustion engines compared to fuel cells for cars. The implied large knowledge stock for internal combustion engines means that there are many ideas to build on and that new ideas can be found in many dimensions. When the R&D activity in internal combustion technology is high, a new idea may benefit many researchers in future periods, i.e. the *spillover size effect* is large. However, decreasing returns to new ideas are present. Decreasing returns does not mean that the best ideas get taken first (i.e. no fishing out), but that the benefit for future R&D is relatively small from a new idea when it is just one more idea in an already large pool of knowledge. In the fuel cell technology, the benefit for researchers from a new idea may be greater than in the internal combustion technology. The reason is that a new idea expands the future research possibilities relatively more when the knowledge stock is small, i.e. the *spillover depletion effect* is smaller in the immature technology.

3 The model

In order to analyze the relative undersupply of R&D in an emerging technology compared to R&D in a mature technology, I develop a partial model of the private enterprises in an economy, with final goods producers, intermediate goods producers, and patents (ideas) producers.

The final goods industry manufactures output (e.g. transportation services) and is characterized by productivity increase from an expansion of the number of available capital varieties (Romer, 1990). In the intermediate goods industry, firms buy patents from one of the R&D industries. A patent gives a firm an exclusive right to produce one type of capital variety. The intermediate goods firms engage in monopolistic competition and deliver capital varieties to the manufacturer. There are two R&D industries, one emerging and one mature. These R&D industries produce new patents in their respective technology field; small scale emerging technology (e.g. hydrogen-based car engines) and large scale mature technology (e.g. internal combustion car engines). The accumulated production of patents gives rise to two different knowledge stocks, which lower the cost of patent production in the respective technologies. The maturity of a technology is defined by the amount of patents in the technology, i.e. the size of the knowledge stock.

I make two assumptions in order to focus on the role of maturity in the allocation of resources to R&D in different technologies. Firstly, I assume that it does not matter for the final goods industry whether capital variants are produced by one technology or the other. The final goods industry gets the same productivity increase from a capital variant based on the emerging technology as one based on the mature technology. By making this assumption I manage to isolate the effect that the maturity of the technologies, through the knowledge stocks, has on the investment decision in the R&D industries.

Secondly, I assume that the total allocation of resources dedicated to R&D is given. I disregard the allocation between final goods production and patent production since the focus of this paper is on the relative undersupply of the two technologies. In general, R&D is undersupplied by private firms in the model presented here, and subsidies should be given to internalize knowledge spillovers. The undersupply of R&D depends on the difference between the social and the private rates of return from R&D, where the rates of return give the social and private allocation of resources. However, in this paper I do not study the undersupply of R&D per se, but the difference in the social and the private allocation between the two technologies. Both technologies are undersupplied, but the question I raise is whether one is more undersupplied than the other.

If one technology is more undersupplied then R&D in that technology should be subsidized more than R&D in the other.

The role of maturity can be studied in a more elaborate model where the production of patents is directed towards two different final goods. This can add a market size effect and a price effect (Acemoglu, 2002) as well as market imperfections other than spillovers to the allocation decision between R&D in the emerging and the mature technology. However, the focus of this paper is to study whether maturity on its own is a valid argument for differentiating R&D policy. Optimal R&D policy follows from the externalities in the research process, and if other externalities are included, e.g. emission externalities, these should be targeted by separate policies.

3.1 Competitive equilibrium

In the competitive equilibrium private firms maximize profits without taking into account the externalities arising from knowledge spillovers. In this section I derive the private allocation of resources to the two R&D industries, when the government does not intervene in the market.

3.1.1 Final goods industry

The final goods industry manufactures output with the following production function:

$$Y_t = L_{Y,t}^{1-\alpha} \int_0^{A_t} x_{i,t}^{\alpha} di : \alpha \in (0,1), \tag{5}$$

where $L_{Y,t}$ is labor input, $x_{t,i}$ is input of capital variant i, and A_t is total knowledge stock. The total knowledge stock is given by $A_t = A_{e,t} + A_{m,t}$, where $A_{e,t}$ is knowledge stock in the emerging technology, e, and $A_{m,t}$ is knowledge stock in the mature technology, m. The knowledge stocks represent the amount of patents available. More patents correspond to more capital variants and increased productivity. Time, t , is suppressed in the rest of the paper where not otherwise noted. Y is sold for a numeraire price equal to 1.

A representative firm hires labor at rate w_Y and buys capital variants at price p_i, takes prices as given, and solves

$$\max_{L_Y, x_i} : L_Y^{1-\alpha} \int_0^A x_i^{\alpha} di - w_Y L_Y - \int_0^A p_i x_i di.$$

The maximization problem gives the following first order conditions:

$$L_y = (1-\alpha)\frac{Y}{w_Y} \tag{6}$$

$$x_i = (\frac{\alpha}{p_i})^{\frac{1}{(1-\alpha)}} \frac{(1-\alpha)Y}{w_Y} : \vee i, \tag{7}$$

where I have substituted back for Y from (5). Equation (6) gives the demand for labor in the final goods industry and equation (7) is the demand for capital variant i.

3.1.2 Intermediate goods industry

The firms in the intermediate goods industry buy one patent each from one of the R&D industries. The patent is a fixed cost for the firm and gives an exclusive right to produce a capital variant based on that patent. They transform capital goods into intermediate goods in a one to one ratio and sell to the final goods industry in a monopolistic competition. The production technology (or rather the capital-conversion) is the same for all intermediate firms. There is free entry into this industry in the sense that anyone can bid for a patent and produce a capital variety. An intermediate good firm solves the following problem:

$$\max_{x_i} : p(x_i)x_i - rx_i,$$

where r is interest rate on capital, i.e. cost of production, and $p(x_i)$ is the inverse demand for capital variety i from the final goods sector. The first order condition is

$$\frac{\partial p_i}{\partial x_i}\frac{x_i}{p_i} + 1 = \frac{r}{p_i}, \tag{8}$$

where $\frac{\partial p_i}{\partial x_i}\frac{x_i}{p_i}$ is equal to the negative inverse price elasticity from equation (7), $\alpha - 1$. The price elasticity is equal for all for capital variants. Thus the price for the variants is equal for all i, $p_i = p = \frac{r}{\alpha}$, where $\frac{1}{\alpha}$ can be interpreted as a markup factor.

The equal price together with demand from equation (7) imply that the demands for all capital variants are equal, $x_i = x$, and that the instantaneous profit π is the same for all the intermediate goods firms:

$$\pi = px - rx = (1-\alpha)px. \tag{9}$$

The instantaneous profit for all intermediate goods firms is the same since they have the same marginal costs and face the same elasticity of demand for their products.

3.1.3 R&D industries

There are two industries producing patents, one in the emerging technology, e , and one in the mature technology, m. The production of patents is given by the following production function:

$$\dot{A}_j = \nu L_j^{\lambda} A_j^{\phi} : j = e, m, \tag{10}$$

where $0 < \lambda < 1$ and $0 < \phi < 1$. The only difference between producing patents in the two technologies follows from the knowledge stocks. The initial knowledge stock is smaller in the emerging technology than in the mature technology, i.e. $A_{e,0} < A_{m,0}$. The firms do not take into account that their patent production influences other firms' productivity, i.e. they take $\bar{\nu}_j = \nu A_j^{\phi}$ as given. There is free entry into the R&D industries. A representative firms solves

$$\max_{L_j} : P_j \bar{\nu}_j L_j^{\lambda} - w_A L_j : j = e, m,$$

taking the price of the patents P_j and the wage rate w_A as given. The maximization problem gives the following first order condition in the two industries:

$$P_j \bar{\nu}_j \lambda L_j^{\lambda - 1} = w_A : j = e, m. \tag{11}$$

The first order condition gives the resource allocation to R&D in the emerging and the mature technology. This condition can be interpreted as a free entry condition as firms establish in both industries until marginal revenue equates marginal costs.

That the intermediate goods firms have the same profits whichever technology they supply to the final goods industry implies that the price of a patent is equal in the two technologies, i.e. $P_e = P_m = P$ (see Appendix A). The equality of patent prices implies that the only source of difference in the production of ideas between the two R&D industries emanates from the knowledge stocks. The difference in knowledge stocks leads to the following proposition:

Proposition 1 *In the competitive equilibrium labor allocation and patent production are always higher in the mature R&D industry than in the emerging R&D industry.*

Proof. Rearranging (11) gives $L_j = (\frac{\lambda \nu P A_j^{\phi}}{w_A})^{\frac{1}{1-\lambda}}$. $A_m > A_e \Rightarrow L_m > L_e$, when $\phi \geq 0$ and $\lambda < 1$. This together with equation (10) gives $\dot{A}_m > \dot{A}_e$. ■

Proposition 1 follows from the mature technology having a larger knowledge stock than the emerging. A larger knowledge stock implies higher patent productivity for given output elasticities. Since factors other than productivity, i.e. patent price and wage rate, are equal in the two R&D industries, firms always invest more in the mature R&D industry.

Total labor dedicated to R&D in the economy L_A is given by assumption, i.e. $L_A = L_e + L_m$. This assumption can be understood as a division of the labor force into two separate markets; one market for

R&D with a highly specialized workforce and one for other activities, L_Y. The assumption implies that when one type of R&D increases, e.g. from a subsidy, the other type of R&D is crowded out. In order to compare the competitive allocation with the socially optimal allocation, which is derived in the next section, I normalize L_A to one and define the allocation ratio between the two technologies $\frac{1-L_m}{L_m}$. From equation (11) we have that the values of the marginal products in the mature and the emerging R&D industry equate in equilibrium:

$$P\bar{\nu}_m\lambda(L_m^{eq})^{\lambda-1} = P\bar{\nu}_e\lambda(1-L_m^{eq})^{\lambda-1}$$

$$\Leftrightarrow$$

$$\frac{1-L_m^{eq}}{L_m^{eq}} = (\frac{A_e}{A_m})^{\frac{\phi}{1-\lambda}}, \tag{12}$$

where I use $\bar{\nu}_j = \nu A_j^{\phi}$ and L_m^{eq} is the labor allocation to R&D in the mature technology in the competitive equilibrium. We see that the private allocation ratio between the two technologies, $\frac{1-L_m^{eq}}{L_m^{eq}}$, is given by the knowledge stock ratio, $\frac{A_e}{A_m}$[3].

The allocation ratio in equation (12) reproduces Proposition 1. This allocation ratio highlights that it is only the maturity of the technologies that matters for the private firms' allocation of labor between the two technologies.

3.2 The social optimal equilibrium

In this section I solve a simplified social planner problem to find the social optimal (efficient) allocation of labor between the mature and the emerging R&D industry.

The social planner maximizes output over the time period by allocating labor between the two R&D industries. Final goods production is given by $Y_t = L_{Y,t}^{1-\alpha}\int_0^{A_t} x_{ti}^{\alpha}di$ and total capital is given by $\int_0^{A_t} x_t dt = K_t$. The symmetry of capital goods implies that $x_i = \frac{K}{A}$ and the final goods production function can be written $Y = A^{1-\alpha}K^{\alpha}L_Y^{1-\alpha}$. The labor allocation to final goods production is given by assumption. Thus, maximizing output is equivalent to maximizing the total knowledge stock.

[3]There is no investment in the emerging technology if there are constant returns to resource input, $\lambda = 1$ (i.e. no stepping on toes), since the marginal product of resource input in R&D production is always greater for the mature technology in this case.

The social planner problem is

$$\max_{L_m} : \int_0^\infty (A_m + A_e)e^{-rt}dt : L_m \in (0,1)$$

subject to

$$\begin{aligned}\dot{A}_m &= L_m^\lambda A_m^\phi \\ \dot{A}_e &= (1-L_m)^\lambda A_e^\phi\end{aligned}$$

given the initial stocks of knowledge $A_{m,0}$ and $A_{e,0}$, and the discount rate r, i.e. the interest rate. The first order condition gives the social allocation rule between the two R&D industries (see Appendix B):

$$\frac{1-L_m^*}{L_m^*} = \frac{\mu_e \dot{A}_e}{\mu_m \dot{A}_m}, \tag{13}$$

where L_m^* is the social optimal allocation of labor to R&D in the mature technology, μ_m is the shadow value of patents in the mature technology, and μ_e is the shadow value of patents in the emerging technology[4]. As in the competitive equilibrium equation (13) implies that the value of the marginal products equates in equilibrium. The social allocation ratio can be rewritten:

$$\frac{1-L_m^*}{L_m^*} = (\frac{\mu_e}{\mu_m})^{1/(1-\lambda)}(\frac{A_e}{A_m})^{\phi/(1-\lambda)}. \tag{14}$$

In the social optimum, the allocation ratio is dependent on the relative shadow values of patents in addition to the knowledge stocks. The allocation of labor to the emerging technology can only be larger than that to the mature technology if the shadow value is larger in the emerging technology, since the knowledge stock is larger in the mature technology on a transition path.

The shadow values of patents can be interpreted through two components. Firstly, the shadow values give the intratemporal value of new patents for current period final goods production. The intratemporal value of patents is higher than the price of patents in the competitive equilibrium due to the surplus appropriability problem. The firms in the intermediate goods industry engage in monopolistic competition and are unable to appropriate the entire "consumer surplus" from the goods they sell. Hence, the price of patents facing the R&D firms is lower than the socially optimal price, since the former price is equal to the present value

[4]This simple, partial maximization problem gives the same allocation rule between the two R&D industries as a full social planner problem where the resource allocation between final goods and R&D production is not given, see Appendix C.

of the profits of the intermediate goods firms. However, the monopolistic competition in this paper is symmetric between firms delivering capital variants in the emerging technology and the mature technology. Hence, even though the surplus appropriability problem implies that R&D is undersupplied in the competitive equilibrium, the problem does not imply that one technology should be subsidized more than the other.

Secondly, the shadow values give the intertemporal value of new patents reducing costs of future patent production. The intertemporal value represents the spillover effect, which may vary between the two technologies according to the *spillover size effect* and the *spillover depletion effect*, and is the reason to diversify R&D subsidies in this paper.

4 The relative undersupply of the technologies

The relative undersupply of R&D in the two technologies is found by comparing the private allocation of labor with the social allocation. By comparing equation (12) with equation (14) it is clear that the difference between the private and the social allocations follows the shadow values. In the competitive equilibrium, the maturity of the technologies matters for the allocation of labor because this influences the productivity in the R&D industries. In the social equilibrium, the maturity of the technologies has an additional intertemporal effect because the maturity also matters for the spillovers that reduce costs of producing patents in later periods. The following proposition states the relationship between the shadow values and the allocation of labor in the private and the social equilibrium:

Proposition 2 *On the relative allocation of labor to the R&D industries:*

1. *If the shadow values of patents equate, $\mu_m = \mu_e$, the competitive allocation is the same as the socially efficient allocation.*

2. *If the shadow value of patents is larger for one of the technologies, the competitive equilibrium undersupplies R&D in that technology more than R&D in the other technology.*

When the competitive equilibrium undersupplies one type of R&D more, it is socially efficient to subsidize that type of R&D more. This social efficiency argument is stated in the following corollary:

Corollary 3 *If the shadow value of patents is larger for one of the technologies, the government should subsidize R&D in that technology more than R&D in the other technology.*

4.1 Are the shadow values equal?

Different maturities in technologies is not an argument for a differentiated R&D policy if the shadow values of patents are equal. The development of the shadow values follows from the costate equations (see Appendix B):

$$\dot{\mu}_m = \mu_m r - \mu_m \phi \frac{\dot{A}_m}{A_m} - 1 \tag{15}$$

$$\dot{\mu}_e = \mu_e r - \mu_e \phi \frac{\dot{A}_e}{A_e} - 1. \tag{16}$$

The development of a shadow value is dependent on the shadow value and the growth rate of the technology in addition to parameter values. The developments of the shadow values are only equal if both the shadow values and the growth rates of the technologies are equal. For the shadow values and the growth rates of the technologies to be equal, the knowledge stocks must also be equal. This relationship between the shadow values and the knowledge stocks is stated in the following proposition:

Proposition 4 *The shadow values of patents are equal if, and only if, the knowledge stocks are equal or* $\phi = 1 - \lambda$.

Proof. If μ_m is to be equal to μ_e, then $\dot{\mu}_m$ must be equal to $\dot{\mu}_e$. It then follows from (15) and (16) that $\mu_m = \mu_e$ and $\dot{\mu}_m = \dot{\mu}_e$ is only valid when $\frac{\dot{A}_m}{A_m} = \frac{\dot{A}_e}{A_e}$. $\frac{\dot{A}_m}{A_m} = \frac{\dot{A}_e}{A_e}$ inserted into equation (13) gives $\frac{1-L_m^*}{L_m^*} = \frac{\mu_e A_e}{\mu_m A_m}$. Inserting $\mu_m = \mu_e$ in equation (14) gives $\frac{1-L_m^*}{L_m^*} = (\frac{A_e}{A_m})^{\phi/(1-\lambda)}$. For both statements of L_m^* to be simultaneously true, A_m must be equal to A_e, except for the knife edge case $\phi = 1 - \lambda$. ■

Proposition 4 implies that the allocation of labor to R&D in the emerging and the mature technology is not the same in the competitive and the social equilibrium, except when the knowledge stocks are equal. In other words, innovation externalities caused by knowledge spillovers differ between technologies of different maturity.

There is a case when the knowledge stocks are equal; the balanced growth path. Along a balanced growth path all variables grow at constant rates. The constant growth rates imply that the relative value of new patents has to be constant and that $\frac{\dot{A}_m}{A_m}$ is equal to $\frac{\dot{A}_e}{A_e}$ (see Appendix D), which in turn implies that the knowledge stocks are equal. The following proposition states the equality of shadow values and knowledge stocks on the balanced growth path:

Proposition 5 *On a balanced growth path both the shadow values of patents and the knowledge stocks are equal in the two technologies.*

Proof. i)On a balanced growth path the relative value of patents, $\frac{\mu_e A_e}{\mu_m A_m}$, is constant. Using that $\frac{\dot{A}_m}{A_m} = \frac{\dot{A}_e}{A_e}$ together with (15) and (16), $\frac{\mu_e A_e}{\mu_m A_m}$ is only constant when μ_m is equal to μ_e (see Appendix D)

ii) When μ_m is equal to μ_e Proposition 4 implies $A_m = A_e$, except for the knife edge case $\phi = 1 - \lambda$. ■

Proposition 2 together with Proposition 5 imply that R&D subsidies should not be diversified on a balanced growth path. In a semi endogenous growth model like the one presented in this paper it is a well known result that subsidies to R&D do not affect the long run-growth rate. However, subsides do affect the long-run level of patents (income) through transitory effects (Jones, 1999). When the economy starts off with different knowledge stocks in the two technologies the economy is on a transition path. The difference in knowledge stocks together with Proposition 4 and Corollary 3 gives the following corollary:

Corollary 6 *On a transition path the government should not subsidize R&D in the two technologies equally.*

The reason that R&D should not be subsidized equally is that the two technologies are not equally undersupplied on a transition path. Corollary 6 implies that the maturity of technologies matters when policymakers are to give socially efficient subsidies to different R&D industries, when the economy is outside the balanced growth path.

Since the relative size of the two knowledge stocks changes over time, there is no reason to believe that the relative undersupply, and thus the difference in optimal subsidies, is constant over time. This argument finds support in Hart (2008). He finds that the gap between social and private returns may vary across time along a transition path.

4.2 Which technology is more undersupplied?

I have shown that the shadow values of patents are unequal outside the balanced growth path. In this section I calculate expressions for the shadow values and derive the condition that determines which of the two technologies is more undersupplied in the competitive equilibrium.

To find expressions for the shadow values I rewrite equation (15) and equation (16) to

$$\dot{\mu}_j + \mu_j f_j(t) = -1 : j = m, e, \tag{17}$$

where $f_j(t) = \phi \frac{\dot{A}_j}{A_j} - r$. Together with the transversality conditions, equation (17) can be solved to find the expressions for the shadow values (see Appendix E for calculations):

$$\mu_j = A_j^{-\phi} e^{rt} \int_t^{\infty} [A_j(z)]^{\phi} e^{-rz} dz : j = m, e. \tag{18}$$

From equation (18) we see that a shadow value is a function of the current knowledge stock and the discounted knowledge stocks of all future periods. The relative shadow value of patents is given by

$$\frac{\mu_e}{\mu_m} = \frac{A_e^{-\phi} \int_t^{\infty} [A_e(z)]^{\phi} e^{-rz} dz}{A_m^{-\phi} \int_t^{\infty} [A_m(z)]^{\phi} e^{-rz} dz}. \tag{19}$$

It is not possible to give a precise interpretation of which shadow value is the larger from equation (19) since we do not know how the knowledge stocks develop over time. However, we can use the relative shadow value to gain an insight into the social allocation of labor and obtain its difference from the private allocation.

Rearranging equation (14) and combining with equation (19) give

$$(\frac{1 - L_m^*}{L_m^*})^{1-\lambda} = \frac{\int_t^{\infty} [A_e(z)]^{\phi} e^{-rz} dz}{\int_t^{\infty} [A_m(z)]^{\phi} e^{-rz} dz}. \tag{20}$$

Equation (20) gives the social allocation of labor between the two R&D industries without the shadow values. As in the private equilibrium, the mature technology gets a larger share of labor in the social equilibrium. This labor allocation result is stated in the following proposition:

Proposition 7 *In the social equilibrium, labor allocation is larger to the mature R&D industry than to the emerging R&D industry, on a transition path.*

Proof. $A_m > A_e \Rightarrow \int [A_m(z)]^{\phi} e^{-rz} dz > \int [A_e(z)]^{\phi} e^{-rz} dz \Rightarrow L_m^* > \frac{1}{2}$ from equation (20). ■

Proposition 7 states that it is always optimal to allocate more labor to the mature R&D industry. This resource allocation rule implies that spillovers in the emerging technology are not large enough to make it optimal to allocate more labor to the emerging R&D industry than to the mature industry.

The difference between the social and the private allocation of labor to the R&D industries can be seen by rearranging equation (12) and subtracting it from equation (20):

$$(\frac{1 - L_m^*}{L_m^*})^{1-\lambda} - (\frac{1 - L_m^{eq}}{L_m^{eq}})^{1-\lambda} = \frac{\int_t^{\infty} [A_e(z)]^{\phi} e^{-rz} dz}{\int_t^{\infty} [A_m(z)]^{\phi} e^{-rz} dz} - (\frac{A_e}{A_m})^{\phi}. \tag{21}$$

Equation (21) highlights the difference between the social and the private allocation; the social allocation depends on the relative size of the knowledge stocks in all periods while the private allocation only depends on the relative size of the knowledge stocks in the current period.

Spillovers in patent production determine which of the two technologies is more undersupplied in the competitive equilibrium. The emerging technology is more undersupplied when spillovers are larger in the emerging technology compared to the mature technology, and vice versa. To find which technology is more undersupplied I analyze the effect on the criterion functional of deviating from the competitive equilibrium (see Seierstad and Sydsæter, 1987). Consider the admissible solution to the planner's maximization problem L_m^{eq} as a *constrained* maximum for the planner. Then define $W(s)$ as the value of the criterion functional for a perturbation of L_m^{eq} defined on an interval given by s (see Appendix F). Then we have that the social gain of perturbating from L_m^{eq} is given by

$$\frac{dW(0)}{ds} = H(A_e, A_m, \mu_e, \mu_m, L_m) - H(A_e, A_m, \mu_e, \mu_m, L_m^{eq}) \gtrless 0$$

$$\Rightarrow$$

$$\mu_m(L_m)^\lambda A_m^\phi + \mu_e(1-L_m)^\lambda A_e^\phi - \mu_m(L_m^{eq})^\lambda A_m^\phi - \mu_e(1-L_m^{eq})^\lambda A_e^\phi \gtrless 0, \quad (22)$$

where A_e, A_m, μ_e and μ_m follows from L_m^{eq}. If the derivative of (22) with respect to L_m in the neighborhood of L_m^{eq} is positive (negative), it is optimal to increase (decrease) L_m from L_m^{eq} with a small integer. The derivative of (22) with respect to L_m in the neighborhood of L_m^{eq} is given by

$$\mu_m\lambda(L_m^{eq})^{\lambda-1}A_m^\phi - \mu_e\lambda(1-L_m^{eq})^{\lambda-1}A_e^\phi \gtrless 0. \quad (23)$$

I use equation (12) and equation (19) to insert for L_m^{eq}, μ_e and μ_m in equation (23):

$$\frac{A_m^{-\phi}\int_t^\infty [A_m(z)]^\phi e^{-rz}dz}{A_e^{-\phi}\int_t^\infty [A_e(z)]^\phi e^{-rz}dz} - 1 \gtrless 0. \quad (24)$$

It follows from equation (24) that the social allocation to the emerging R&D industry is larger than the private allocation if the left hand side of the equation is negative. Further, the sign of the left hand side of equation (24) depends on the growth rates of the knowledge stocks in the competitive equilibrium. This relationship between the relative undersupply of the two technologies and the growth rates of the knowledge stocks is stated in the following proposition:

Proposition 8 *The emerging technology is more (less) undersupplied than the mature technology if the growth rate of A_e in the competitive equilibrium is larger (smaller) than the growth rate of A_m in the competitive equilibrium.*

Proof. $sign[\frac{A_m^{-\phi}\int_t^\infty [A_m(z)]^\phi e^{-rz}dz}{A_e^{-\phi}\int_t^\infty [A_e(z)]^\phi e^{-rz}dz} - 1] = sign[\frac{\int_t^\infty (\frac{A_m(z)}{A_m(t)})^\phi e^{-rz}dz}{\int_t^\infty (\frac{A_e(z)}{A_e(t)})^\phi e^{-rz}dz} - 1]$ which is negative (positive) if $\frac{\dot{A}_e}{A_e}$ is larger (smaller) than $\frac{\dot{A}_m}{A_m}$. In the *constrained* maximum, the growth rates of A_e and A_m follow from L_m^{eq}. ■

Proposition 8 states that the technology that grows faster is more undersupplied in the competitive equilibrium. To understand why the growth rates determine the relative undersupply we can look at equation (21). The relative size of the knowledge stocks change over time when one knowledge stock grows faster than the other. The private allocation only depends on the current period knowledge stocks while the social planner takes into account the knowledge stocks over the whole period. Thus, it is optimal to deviate from the private allocation and increase the allocation to the technology that grows faster.

The growth rates of the knowledge stocks depend on the output elasticity parameters λ and ϕ. This gives a relationship between the relative undersupply of the two technologies and the elasticity parameters that is stated in the following proposition:

Proposition 9 *The emerging technology is more (less) undersupplied than the mature technology if $\lambda + \phi$ is smaller (larger) than one, on a transition path.*

Proof. Using L_m^{eq} it follows that equation (10) and equation (11) give $\frac{\dot{A}_j}{A_j} = kA_j^{\frac{\lambda+\phi-1}{1-\lambda}}$,where $k = v(\frac{w_A}{Pv\lambda})^{\frac{\lambda}{\lambda-1}}$. Since $A_m > A_e$ it follows that $\frac{\dot{A}_e}{A_e} > \frac{\dot{A}_m}{A_m}$ when $\lambda + \phi < 1$ and $\frac{\dot{A}_e}{A_e} < \frac{\dot{A}_m}{A_m}$ when $\lambda + \phi > 1$. ■

Proposition 9 states that the relative undersupply of the technologies is determined by the sum of the output elasticity parameters in R&D production, i.e. the elasticity of scale. The reason is that the elasticity of scale determines the growth rates of the knowledge stocks in the competitive equilibrium. The market outcome gives a larger undersupply of the emerging technology compared to the mature technology when the elasticity of scale is smaller than one. In this case it is optimal to subsidize the emerging R&D industry more than the mature R&D industry. When the elasticity of scale is larger than one it is optimal to subsidize the mature R&D industry more than the emerging R&D industry.

5 Numerical illustration

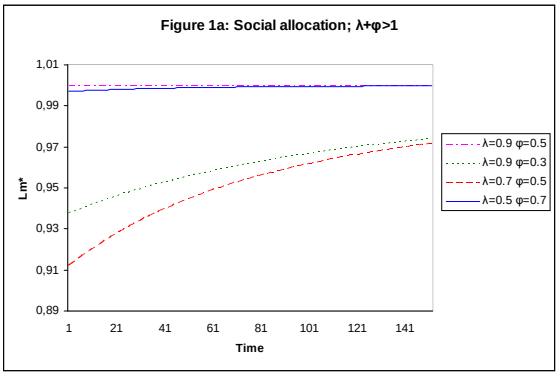
I now turn to numerical simulations to illustrate the difference between the social and the private allocation between the two R&D industries. First I show how the social allocation is dependent on the two output elasticity parameters, λ and ϕ, and how these parameters determine the difference between the social and the private allocation. Then I show the effect of including knowledge spillovers between the two R&D industries.

5.1 Numerical procedure

The simulation model is programmed as a discrete time model over 150 periods. I assume that patents produced in one time period are not included in the current period's knowledge stock. Knowledge accumulates according to $A_{j,t+1} = A_{j,t} + L_{j,t}^{\lambda} A_{j,t}^{\phi}$, $j = m, e$. The initial knowledge stocks are arbitrarily chosen so that $A_{m,0} > A_{e,0}$. Total resources devoted to R&D are given; $1 = L_e + L_m$. In the competitive equilibrium, firms maximize their profits by setting L_m and L_e simultaneously. In the social equilibrium, a social planner maximizes the present value of production by setting L_m and L_e simultaneously.

5.2 The social allocation

The social allocation to the mature R&D industry L_m^* for different output elasticity parameters is given in Figures 1a and 1b:



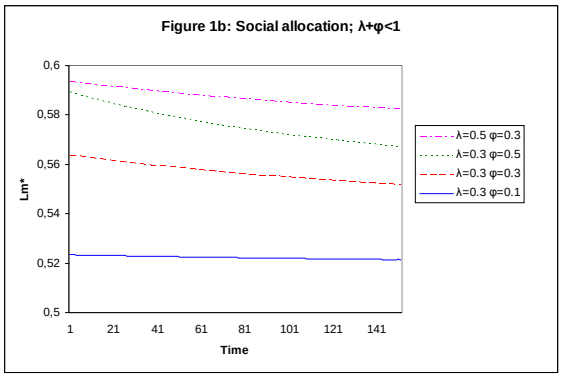
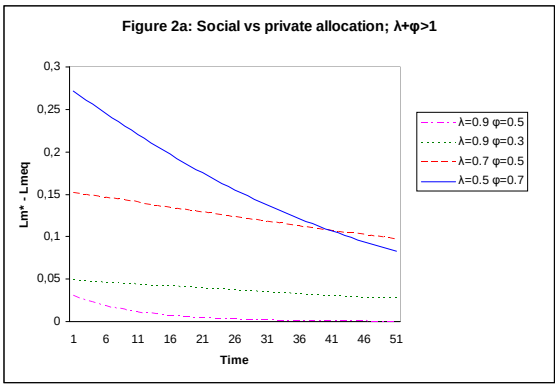


Figure 1a shows the social allocations when $\lambda + \phi > 1$, while Figure 1b shows the social allocations when $\lambda + \phi < 1$. From the figures we see that the social allocation is crucially dependent on the elasticity of scale: L_m^* is slightly below one and increasing when $\lambda + \phi > 1$, and L_m^* is slightly below 0.6 and decreasing when $\lambda + \phi < 1$. The social allocation is constant and equal for all parameter combinations that give $\lambda + \phi = 1$. Proposition 7 is reproduced in the simulations for all parameter values; the social allocation is larger to the mature R&D industry than to the emerging R&D industry.

The reason the output elasticities matter for the allocation between the mature and the emerging R&D industry is that the elasticities give the productivity of patent production; for given knowledge stocks, higher elasticities imply a larger patent production. The patent productivity is larger in the mature technology since the knowledge stock is larger. However, the patent productivity advantage to the mature technology is smaller when the elasticities are lower. The reason is the decreasing returns to both knowledge and resource input in the R&D production function. The decreasing returns imply that when the elasticities are lower it is optimal to allocate more labor to the emerging technology than in the case when the elasticities are higher.

5.3 The difference between the social and private allocation

The difference between the social and the private allocation of labor to the mature R&D industry $L_m^* - L_m^{eq}$ for different parameter values is given in Figures 2a and 2b:



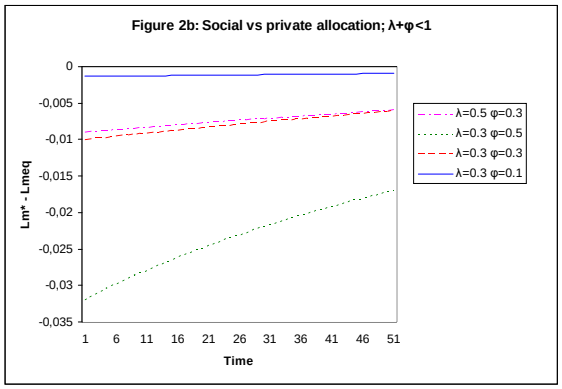


Figure 2a shows the difference between the allocations when $\lambda + \phi > 1$, while Figure 2b shows the difference between the allocations when $\lambda + \phi < 1$. Proposition 9 is reproduced in all simulations. The elasticity of scale determines whether the social or the private allocation to the mature R&D industry is larger; $L_m^* - L_m^{eq}$ is positive when $\lambda + \phi > 1$, and $L_m^* - L_m^{eq}$ is negative when $\lambda + \phi < 1$. Proposition 4 states that the shadow values are equal if $\lambda + \phi = 1$. This result is confirmed in simulations as $L_m^* - L_m^{eq} = 0$ for all parameter combinations that give $\lambda + \phi = 1$. Further, we see that the social allocation approaches the private allocation in the long run.

The difference between the social and the private allocation can be interpreted through the *spillover size* and the *spillover depletion effect*. When $\lambda > 1 - \phi$ the *spillover size effect* is larger than the *spillover depletion effect* and the private firms under invest in the mature R&D industry. The reason is that when one or both output elasticities are high the patent productivity in the mature technology is so large that it is optimal to allocate even more labor to the mature R&D industry than in market solution in order to exploit the initial higher productivity in the mature technology.

When $\lambda < 1 - \phi$ the *spillover depletion effect* dominates and the private firms under invest in the emerging industry. The reason is that when the output elasticities are low there is less patent productivity advantage in the mature technology and the social allocation to the emerging R&D industry is larger than the private allocation due to the diminishing returns to new patents in R&D production.

To sum up, the elasticity of scale in the R&D production function determines which of the two technologies is more undersupplied in the market equilibrium. Thus, it is only a valid argument to give more governmental support to an emerging R&D industry than to a mature R&D industry for a specific range of values for the output elasticities in the R&D production. However, when the elasticity parameters are low we see from Figure 2b that the difference between the social and the private allocation is small. This small difference might indicate that there is no case for subsidizing the emerging R&D industry more than the mature if we account for potential costs of administrating a differentiated R&D policy, e.g. costs of determining which technology is emerging and which is mature.

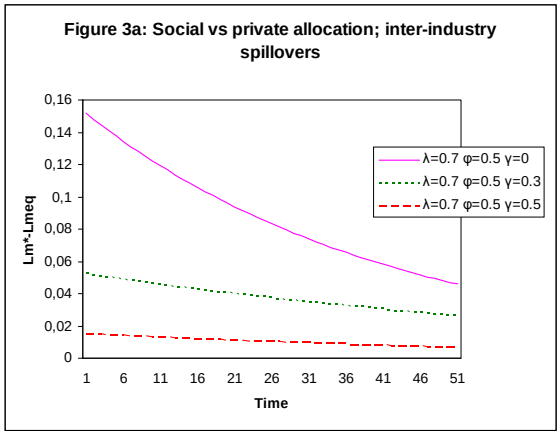
5.4 Knowledge spillovers between the industries

Inter-industry knowledge spillovers may reduce the difference in externalities from R&D in the two technologies. Hence, the rationale for a differentiated R&D policy may diminish. To allow for inter-industry knowledge spillovers I include both of the knowledge stocks in the R&D production functions in the following way:

$$\dot{A}_j = \nu L_j^{\lambda}(A_j + \gamma A_{-j})^{\phi} : j = e, m, \tag{25}$$

where $\gamma \in (0, 1)$ is the inter-industry spillover parameter. A high parameter value implies large inter-industry spillovers, while a low value implies small inter-industry spillovers. Figures 3a and 3b give the difference between the social and the private allocation of labor when there

are knowledge spillovers between the two R&D industries:



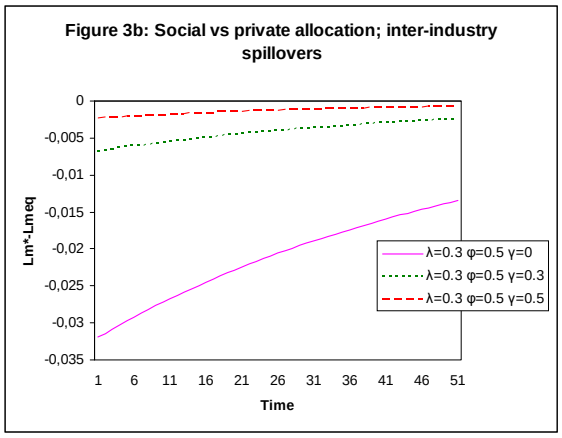


Figure 3a shows the difference between the allocations when $\lambda + \phi > 1$, while Figure 3b shows the difference between the allocations when $\lambda + \phi < 1$. From the figures we see that an increase of the inter-industry spillover parameter reduces the difference between the social and the private allocation. If there are complete knowledge spillovers between the industries, i.e. $\gamma = 1$, there is no reason for the social planner to have a different allocation ratio between the R&D industries than the private firms. However, if the inter-industry spillovers are incomplete, i.e. $\gamma < 1$, the relative undersupply of the technologies follows from the elasticity of scale in the R&D production function.

5.5 Sensitivity analysis

The knowledge stocks are arbitrarily chosen. Different initial knowledge stocks that either change the relative size of the knowledge stocks or only the absolute size of the knowledge stocks do not change how the elasticity parameters determine the social allocation versus the private

allocation to the R&D industries[5].

The interest (discount) rate is also arbitrarily chosen. A higher interest rate narrows the gap between the social allocation and the private allocation as the social planner gives less weight to late periods (see Appendix G, Figure 4). However, different interest rates do not change the threshold value for the elasticity of scale.

6 Discussion and conclusion

An important policy question is whether R&D in new, emerging technologies should be subsidized more than R&D in other more mature technologies. In this paper I analyze if innovation externalities caused by knowledge spillovers from private firms may warrant a differentiation of R&D policy towards technologies of different maturity.

I find that the governmental support for R&D in emerging and mature technologies should not be equal. The reason is that R&D in the two technologies is not equally undersupplied in the market due to differences in their knowledge stocks. Hence, the maturity of the technologies matters when policymakers are to give socially efficient subsidies to different R&D industries.

However, whether the emerging technology or the mature technology is more undersupplied in the market solution depends on the sum of the output elasticities with respect to labor and knowledge in R&D production, i.e. the elasticity of scale. The reason is that the elasticities determine the growth rates of the knowledge stocks in the two technologies. The labor allocation between the R&D industries follows from the relative size of the knowledge stocks. However, in the competitive equilibrium, the labor allocation only depends on the current period knowledge stocks, while the social planner takes into account the knowledge stocks over the whole time period. Hence, it is optimal to deviate from the private labor allocation and increase the allocation to the technology that grows faster. When the elasticity of scale is larger than one the mature technology grows faster and is more undersupplied. Thus, the mature R&D industry should be subsidized more. When the elasticity of scale is smaller than one the result is reversed and the emerging R&D industry should be subsidized more.

The size of the output elasticity parameters is an empirical question. One of the few studies that estimate the parameters in the R&D production function is Gong et al. (2004). They find the output elasticity with respect to knowledge to be about 0.01, while the labor elasticity is insignificant. Another approach is Jones and Williams (2000) where the

[5] Tables from these simulations can be provided by the author upon request.

calibrated ranges of output elasticities all give an elasticity of scale larger than one. Further empirical research is needed to establish significant ranges for the output elasticities.

Moreover, the output elasticity parameters are the same for both R&D industries in this paper. Differences in the parameters may increase the difference in spillovers from R&D in the two technologies and strengthen the case for differentiated subsidies. This is an avenue for future research.

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Appendix

Appendix A - The equality of patent prices

The price of a patent is equal to the present discounted value of the profits that an intermediate goods firm gets from supplying a type of capital variety, $P_j = PDV$. This price follows from an arbitrage argument due to free entry into the intermediate goods industry; if $P_j > PDV$ firms are not established in the industry, and if $P_j < PDV$ more firms will be established in the industry. Firms that want to enter the market bids up the price of a patent until $P_j = PDV$. In other words, the R&D industries extract all the profits from the monopolistic behavior in the intermediate goods industries.

The instantaneous profit for all intermediate goods firms is the same since they have the same marginal costs and face the same elasticity of demand for their products. The instantaneous profit is given by

$$\pi_i = (1-\alpha)\alpha L_Y^{1-\alpha} x_i^{\alpha}, \tag{26}$$

where equation (7) is inserted in equation (9). Note that firms are indexed with i as this will be used in the next step. Integrating equation (26) on both sides over the total knowledge stock A gives

$$A\pi_i = (1-\alpha)\alpha L_Y^{1-\alpha} \int_0^A x_i^{\alpha} di$$

$$\Longleftrightarrow$$

$$\pi = (1-\alpha)\alpha \frac{Y}{A}, \tag{27}$$

where I have inserted from equation (5) in the last step. The present discounted value is equal across all intermediate goods firms and can be written as

$$PDV_t = \int_t^{\infty} e^{-rz}(1-\alpha)\alpha \frac{Y(z)}{A(z)} dz. \tag{28}$$

The price the two R&D industries charge for their patents has to be equal, $P_e = P_m = P$, because PDV_t is equal for all firms regardless of which R&D industry they buy patents from.

Appendix B - The partial socially optimal equilibrium

The autonomous Hamiltonian H is given from the simple, partial social planner problem:

$$H = A_m + A_e + \mu_m L_m^{\lambda} A_m^{\phi} + \mu_e (1 - L_m)^{\lambda} A_e^{\phi},$$

From the Hamiltonian, I get the following first order condition:

$$\frac{\partial H}{\partial L_m} : \mu_m \lambda L_m^{\lambda-1} A_m^{\phi} - \mu_e \lambda (1 - L_m)^{\lambda-1} A_e^{\phi} = 0, \tag{29}$$

which together with the development of the shadow values from the costate equations

$$\dot{\mu}_m = \mu_m r - \mu_m \phi L_m^{\lambda} A_m^{\phi-1} - 1 \tag{30}$$

$$\dot{\mu}_e = \mu_e r - \mu_e \phi (1 - L_m)^{\lambda} A_e^{\phi-1} - 1 \tag{31}$$

and the transversality conditions

$$\lim_{t\to\infty} \mu_m e^{-rt} A_m = 0$$
$$\lim_{t\to\infty} \mu_e e^{-rt} A_e = 0$$

give solution to L_m^*. I rearrange equation (29) to get

$$\frac{1 - L_m^*}{L_m^*} = (\frac{\mu_e}{\mu_m})^{1/(1-\lambda)} (\frac{A_e}{A_m})^{\phi/(1-\lambda)}. \tag{32}$$

Inserting $\dot{A}_m$ and $\dot{A}_e$ in equation (32) gives

$$\frac{1 - L_m^*}{L_m^*} = \frac{\mu_e \dot{A}_e}{\mu_m \dot{A}_m}.$$

Appendix C - The full socially optimal equilibrium

In the full social planner problem, resources are allocated to final goods production as well as to R&D in the emerging and the mature technology, i.e. $L_t = L_{Y,t} + L_{e,t} + L_{m,t}$. Final goods production is given by $Y_t = L_{Y,t}^{1-\alpha} \int_0^{A_t} x_{i,t}^{\alpha} di$ and total capital is given by $\int_0^{A_t} x_t dt = K_t$. Time t is suppressed where not otherwise noted. The symmetry of capital goods

implies that $x_i = \frac{K}{A}$ and the final goods production function can be written $Y = A^{1-\alpha}K^{\alpha}L_Y^{1-\alpha}$. In per capita terms $k = \frac{K}{L}$ and $y = \frac{Y}{L}$, and the final goods production can be written $y = A^{1-\alpha}k^{\alpha}(1 - s_m - s_e)^{1-\alpha}$, where $s_m = \frac{L_m}{L}$ and $s_e = \frac{L_e}{L}$.

Discounted utility U is given by

$$U_t = \int_t^{\infty} L_s u(c_s) e^{-r(s-t)} ds,$$

where $L_t = L_0 e^{nt}$, n is the population growth rate, and $u(c_t)$ is the instant utility of consumption per capita $c = \frac{C}{L}$. The final goods are converted into consumption goods or capital goods in a one-to-one ratio so that capital per capita grows according to $\dot{k} = y - c - (n + \delta)k$, where δ is the depreciation rate of capital.

The social planner problem is

$$\max_{c, s_m, s_e} : \int_t^{\infty} L_0 u(c) e^{-\bar{\rho}t} dt$$

subject to

$$\begin{aligned} \dot{k} &= y - c - (n + \delta)k \\ \dot{A}_m &= \nu s_m^{\lambda} L^{\lambda} A_m^{\phi} \\ \dot{A}_e &= \nu s_e^{\lambda} L^{\lambda} A_e^{\phi} \end{aligned}$$

given L_0, K_0, $A_{m,0}$, $A_{e,0}$, where $\bar{\rho} = r - n$.

The autonomous Hamiltonian H is given by

$$H = u(c) + \mu_k(y - c - (n + \delta)k) + \mu_m(\nu s_m^{\lambda} L^{\lambda} A_m^{\phi}) + \mu_e(\nu s_e^{\lambda} L^{\lambda} A_e^{\phi}),$$

where μ_k is the shadow value of capital, μ_m is the shadow value of patents in the mature technology, and μ_e is the shadow value of patents in the emerging technology. From the Hamiltonian, I get the following first order conditions:

$$\frac{\partial H}{\partial c} : u'(c) - \mu_k = 0$$

$$\frac{\partial H}{\partial s_m} : -\mu_k(1 - \alpha)A^{1-\alpha}k^{\alpha}(1 - s_m - s_e)^{-\alpha} + \mu_m \nu \lambda s_m^{\lambda - 1} L^{\lambda} A_m^{\phi} = 0 \quad (33)$$

$$\frac{\partial H}{\partial s_e} : -\mu_k(1 - \alpha)A^{1-\alpha}k^{\alpha}(1 - s_m - s_e)^{-\alpha} + \mu_e \nu \lambda s_e^{\lambda - 1} L^{\lambda} A_e^{\phi} = 0, \quad (34)$$

which together with the development of the shadow values from the costate equations

$$\dot{\mu}_k = \mu_k\bar{\rho} - \mu_k(\alpha A^{1-\alpha}k^{\alpha-1}(1-s_m-s_e)^{1-\alpha} - (n+\delta))$$

$$\dot{\mu}_m = \mu_m\bar{\rho} - \mu_k\sigma A^{-\alpha}k^{\alpha}(1-s_m-s_e)^{1-\alpha} - \mu_m\phi\nu s_m^{\lambda}L^{\lambda}A_m^{\phi-1} \tag{35}$$

$$\dot{\mu}_e = \mu_e\bar{\rho} - \mu_k\sigma A^{-\alpha}k^{\alpha}(1-s_m-s_e)^{1-\alpha} - \mu_e\phi\nu s_e^{\lambda}L^{\lambda}A_e^{\phi-1} \tag{36}$$

and the transversality conditions

$$\lim_{t\to\infty} \mu_k e^{-\bar{\rho}t}K = 0$$
$$\lim_{t\to\infty} \mu_m e^{-\bar{\rho}t}A_m = 0$$
$$\lim_{t\to\infty} \mu_e e^{-\bar{\rho}t}A_e = 0$$

give solutions to c, s_m and s_e. The Hamiltonian is concave in c, s_m, s_e and in A_m, A_e, thus the necessary condition is satisfied (Arrow condition).

Rearrange equation (33) and equation (34) and insert for y, $\dot{A}_m$, and $\dot{A}_e$ to get

$$\mu_k = \frac{\mu_m\lambda\dot{A}_m(1-s_m-s_e)}{s_m(1-\alpha)y} \tag{37}$$

$$\mu_k = \frac{\mu_e\lambda\dot{A}_e(1-s_m-s_e)}{s_e(1-\alpha)y}. \tag{38}$$

By combining equation (37) and equation (38) I get

$$\frac{s_e^*}{s_m^*} = \frac{\mu_e\dot{A}_e}{\mu_m\dot{A}_m}, \tag{39}$$

which is the same resource allocation rule as in the main text, equation (13), where $s_e^* = 1- s_m^*$ when I disregard the allocation to final goods production, and $s_m^* = L_m^*$ when $L_e + L_m = 1$.

Appendix D - The balanced growth path

On the balanced growth path, the allocation between mature and emerging R&D s_m and the growth rates of the knowledge stocks g_{A_j} have to be constant. I find the growth rate of knowledge by taking logs and deriving the patent production function on the balanced growth path:

$$\begin{aligned}
\dot{A}_j &= L_j^{\lambda} A_j^{\phi} : j = m, e \\
&\Leftrightarrow \\
\ln \frac{\dot{A}_j}{A_j} &= \lambda \ln L_j + (\phi - 1) \ln A_j \\
&\Rightarrow \\
g_{A_j} &= 0.
\end{aligned}$$

The growth rate is zero on the balanced growth path because total labor input to R&D does not grow (in the full socially optimal equilibrium the growth rate is given by $g_{A_j} = \frac{\lambda n}{1-\phi}$).

On a balanced growth path the relative value of patents, $\frac{\mu_e A_e}{\mu_m A_m}$, is constant. To show that the shadow values are equal on the balanced growth path I first define the relative shadow values $\chi = \frac{\mu_e}{\mu_m}$ and derive this with respect to time:

$$\frac{\dot{\chi}}{\chi} = \frac{\dot{\mu}_e}{\mu_e} - \frac{\dot{\mu}_m}{\mu_m}. \tag{40}$$

Insert from equation (16) and equation (15) in equation (40) to get

$$\frac{\dot{\chi}}{\chi} = \frac{1}{\mu_m} - \frac{1}{\mu_e} + \phi \frac{\dot{A}_m}{A_m} - \phi \frac{\dot{A}_e}{A_e}. \tag{41}$$

Then take the log of $\frac{\mu_e A_e}{\mu_m A_m}$ and derive this with respect to time:

$$\begin{aligned}
\ln \chi \frac{A_e}{A_m} &= \ln \chi + \ln A_e - \ln A_m \\
&\Rightarrow \\
0 &= \frac{\dot{\chi}}{\chi} + \frac{\dot{A}_e}{A_e} - \frac{\dot{A}_m}{A_m},
\end{aligned} \tag{42}$$

where $\frac{\partial}{\partial t}(\ln \chi \frac{A_e}{A_m}) = 0$ on the balanced growth path. Inserting for $\frac{\dot{\chi}}{\chi}$ from equation (42) into equation (41) gives

$$0 = \frac{1}{\mu_m} - \frac{1}{\mu_e} + \phi \frac{\dot{A}_m}{A_m} - \phi \frac{\dot{A}_e}{A_e} + \frac{\dot{A}_e}{A_e} - \frac{\dot{A}_m}{A_m}. \tag{43}$$

Since the growth rates of knowledge are equal on the balanced growth path (and in this case zero), equation (43) shows that the shadow values are equal on the balanced growth path. When μ_m is equal to μ_e Proposition 5 implies $A_m = A_e$. Thus, equation (14) implies that $L_m = 1/2$ on the balanced growth path.

Further, on the balanced growth path equation (16) and equation (15) simplifies to

$$\dot{\mu} = \mu r - 1, \tag{44}$$

where $j = m, e$ is suppressed since the shadow values are equal. I solve equation (44) to find the shadow values on the balanced growth path:

$$\mu = \frac{1}{r} + Ce^{rt}, \tag{45}$$

where C is a constant. This constant can be found by utilizing the transversality condition, $\lim_{t\to\infty} \mu e^{-\bar{\rho}t} A = 0$. I substitute for $\mu = \frac{1}{r} + Ce^{rt}$ in the transversality condition to get

$$\lim_{t\to\infty} (\frac{e^{-rt}}{r} + C)A = 0 \tag{46}$$

which is only valid for $C = 0$. Hence, I get the shadow values of patents on balanced growth path:

$$\mu_j = \frac{1}{r} : j = m, e. \tag{47}$$

Appendix E - Calculation of the shadow values

I rewrite equation (15) and equation (16) to

$$\dot{\mu}_j + \mu_j f_j(t) = -1 : j = m, e, \tag{48}$$

where $f_j(t) = \phi \frac{\dot{A}_j}{Aj} - r$. I suppress $j = m, e$ in the following and define $F(t) = \int f(t)dt = \int \phi \frac{\dot{A}}{A} dt + \int r dt = \int \phi \frac{dA}{A} + \int r dt = \phi \ln A - rt$. I multiply both sides of equation (48) by $e^{F(t)}$ and derive with respect to time to get

$$\frac{\partial}{\partial t}(\mu e^{F(t)}) = -e^{F(t)}$$

$$\Longrightarrow$$

$$\mu e^{F(t)} = \int_{\infty}^{t} -e^{F(z)} dz + C, \tag{49}$$

where C is a constant. Inserting for $e^{F(t)} = A^{\phi} e^{-rt}$ equation (49) can be written

$$\mu = A^{-\phi} e^{rt} (C - \int_{\infty}^{t} A^{\phi} e^{-rz} dz). \tag{50}$$

We can find the value of C by using the balanced growth rate and the transversality condition. First, I expand equation (50) by using

$\int A^{\phi}e^{-rt}dt = -\frac{1}{r}A^{\phi}e^{-rt} + \int \frac{\phi}{r}\frac{\dot{A}}{A}A^{\phi}e^{-rt}dt$:

$$\mu = A^{-\phi}e^{rt}C + \frac{1}{r} - A^{-\phi}e^{rt}\int_{\infty}^{t}\frac{\phi}{r}\frac{\dot{A}}{A}A^{\phi}e^{-rz}dz. \tag{51}$$

Then on the balanced growth path we have $\frac{\dot{A}}{A} = 0$ which implies that equation (51) simplifies to

$$\mu = A^{-\phi}e^{rt}C + \frac{1}{r}. \tag{52}$$

Lastly I utilize the transversality condition, $\lim_{t\to\infty}\mu e^{-rt}A = 0$. By substituting for $\mu = A^{-\phi}e^{rt}C + \frac{1}{r}$ in the transversality condition I get

$$\lim_{t\to\infty} A^{1-\phi}C + \frac{e^{-rt}A}{r} = 0, \tag{53}$$

which is only valid for $C = 0$. Inserting for $C = 0$ in equation (52) I get that on the balanced growth path the shadow values of patents are equal for both technologies $j = m, e$ and is given by

$$\mu = \frac{1}{r}, \tag{54}$$

which is the same value as shown in Appendix D.

I get the shadow values outside the balanced growth path by inserting $C = 0$ in equation (50):

$$\mu = A^{-\phi}e^{rt}\int_{t}^{\infty} A^{\phi}e^{-rz}dz. \tag{55}$$

Appendix F - Perturbation of the allocation

Following Seierstad and Sydsæter (1987), page 221, I let $[A_e, A_m, L_m^{eq}]$ be an admissible solution to the maximization problem. $[\mu_e, \mu_m]$ follows from the costate equations. L_m^{eq} takes a perturbated value L_m near τ, where $\tau \in [t_0, t_1]$ is a subinterval of the time horizon. L_m is a constant on some interval $E(s)$, where $E(s) = [\tau, \tau + s)$ if $s \geq 0$, $E(s) = (\tau + s, \tau]$ if $s \leq 0$. The value of the criterion function for a perturbation of L_m^{eq} to the left or right of τ is defined as $W(s)$.

Appendix G - Interest rate sensitivity

Figure 4 shows the difference between the social and the private allocation of labor to the mature R&D industry for different interest rates. The elasticity parameters are constant in the simulations; $\lambda = 0.7$ and $\phi = 0.5$.

Figure 4: Social vs private allocation; λ=0.7, φ=0.5