

Transitional Dynamics in an Endogenous Growth Model with Heterogeneous Consumption Goods*

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May 21, 2009

Abstract

We analyze the transitional dynamics of an endogenous growth model with heterogeneous consumption goods. In this model, convergence is driven by two different forces: the diminishing returns to capital and the growth of the relative price between physical and human capital. Because this second force arises only when heterogeneous consumption goods are introduced, the transitional dynamics in this model exhibits two interesting differences with respect to the transitional dynamics in the standard two-sector growth model with a unique consumption good. First, the initial growth effect of a shock in one of the capital stocks may be the opposite to the one obtained in a model with a unique consumption good. Second, the consumption growth rate exhibits a non-monotonic behavior along the transition when the two forces have opposite growth effects.

JEL classification codes: O41, O47.

Keywords: endogenous growth, consumption growth, transitional dynamics.

*Financial support from the Spanish Ministry of Education and Science and FEDER through grants SEJ2006-03879, SEJ2006-05441, and ECO2008-02752; the Generalitat of Catalonia through the Barcelona Economics program (XREA) and grants SGR2005-00447 and SGR2005-00984.

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1. Introduction

In this paper, we study the transitional dynamics of a multi-sector growth model where consumers utility depends on the consumption of heterogenous goods. As the main difference with the existing literature is the introduction of several consumption goods, we will focus our analysis of the transitional dynamics on the path followed by the rate of growth of consumption expenditure.

Several papers have analyzed the patterns of growth when either there is a unique consumption good (see Caballé and Santos, 1993; Lucas, 1988; Mulligan and Sala-i-Martin, 1993; Uzawa, 1965) or there are several consumption goods and a unique capital stock (see Echevarria, 1997; Kongsamunt, Rebelo and Xie, 2001; Laitner, 2000; Ngai and Pissariades, 2004; and Steger, 2006). In these models, consumption growth depends on the interest rate and convergence is thus determined only by the diminishing returns to capital. In contrast, we show that in a growth model with heterogenous consumption goods and heterogeneous capital stocks, consumption growth depends on the interest rate and also on the growth of the relative capital prices. Therefore, convergence is governed by two different forces: the diminishing returns to capital and the growth of relative prices. Our aim is to show how this second force modifies the pattern of consumption growth.

We analyze a three sector growth model in which two consumption goods are produced using two capital stocks: physical and human capital. The different sectors produce using a constant returns to scale technology and the only difference between the technologies is the intensity of physical capital. In this model, we show that the growth effect of an increase in the relative price of human capital in terms of physical capital depends on the Edgeworth elasticity and on the capital intensity ranking among sectors. To gain some intuition about this result, suppose that the relative price of human capital in terms of physical capital increases, which makes human capital relatively more costly than physical capital. Then, the consumption good produced in the physical capital intensive sector becomes less costly and the price of this consumption good decreases. The effect on consumption growth of a reduction in the price of this consumption good depends on the sign of the Edgeworth elasticity. In fact, we show that when the two consumption goods are Edgeworth substitutes (complementaries) a reduction in the price of this consumption good decreases (increases) the growth of consumption expenditure.

The other force driving the transition is the interest rate. The effect of the interest rate on consumption growth is measured by the intertemporal elasticity of substitution (*IES*).

As in any multi-sector growth model with two capital stocks, the transitional dynamics will be governed by the imbalances between these two capital stocks. However, the existence of two different forces driving the transition yields two interesting differences with respect to the transitional dynamics obtained in the standard growth model with a unique consumption good. First, in growth models with a unique consumption good, convergence in the consumption growth rate occurs from below (above) if the initial value of the ratio of physical to human capital is larger (smaller) than its stationary value. We show that this behavior may be reversed under heterogenous consumption goods. In particular, we provide a condition that implies

that convergence is from above when the initial value of the ratio of capitals is larger than its stationary value and from below otherwise. Interestingly, when this condition is satisfied, the initial effect on consumption growth of a shock in one of the capital stocks will be the opposite to the one obtained in a model with a unique consumption good. As an example, consider an economy suffering a negative shock in human capital. Then, if there is a unique consumption good, this economy will suffer a reduction in consumption growth. In contrast, in our model with heterogeneous consumption, the economy will display an increase in consumption growth.

Second, while the growth rate of consumption exhibits a monotonic behavior in models where the only force governing the transition are the diminishing returns to capital, it may exhibit a non-monotonic behavior in our model. Alvarez-Cuadrado et al. (2004) mention evidence of non-monotonic behavior of the consumption growth rate. Steger (2000), among others, has explained this non-monotonic behavior with the introduction of a minimum consumption that makes preferences non-homothetic. In contrast, in our model this non-monotonic behavior is explained by the presence of the aforementioned two different forces driving the transitional dynamics. In fact, the growth rate exhibits a non-monotonic behavior when these two forces have opposite growth effects.

The two differences we have just mentioned imply that the patterns of growth along the transition strongly depend on the value of the parameters in our model with heterogeneous consumption goods. We simulate the economy in order to show the transition of the growth rate. On the one hand, we show that in the simulated economy the growth rate exhibits a non-monotonic convergence. On the other hand, the sign of the growth effects of a shock in one of the capital stocks depend on the value of the *IES*.

We also use the simulated model to see the growth effects of a permanent technological shock that reduces the total factor productivity. We show that the growth effects of this shock also depend on the value of the *IES*. When this elasticity is sufficiently low, this technological shock implies an initial positive growth effect in our economy with heterogeneous consumption goods, whereas it implies a negative growth effect in the economy with a unique consumption good. Obviously, this result has interesting implications for the business cycle. In particular, it shows that the short run growth effects of technological shocks depend on how relative prices are modified.

The paper is organized as follows. Section 2 outlines the model and characterizes the balanced growth path equilibrium. Section 3 studies the transitional dynamics and presents the main results. Conclusions are summarized in Section 4 and the Appendix contains the proofs.

2. The economy

Let us consider a three-sector growth model in which the output in each sector is obtained from combining two types of capital, k and h , which we denote physical and human capital, respectively. The first sector produces a commodity y using the following production function:

$$y = A (s_y k)^\alpha (u_y h)^{1-\alpha} = A u_y h z_y^\alpha,$$

where s_y and u_y are, respectively, the shares of physical and human capital allocated to this sector, $z_y = s_y k / u_y h$ is the physical to human capital ratio, $A > 0$ is the sectoral total factor productivity (*TFP*), and $\alpha \in (0, 1)$ measures the intensity of physical capital in this sector. We assume that this sector produces manufactures and that the commodity y can be either consumed or added to the stock of physical capital. The law of motion of the physical capital stock is thus given by

$$\dot{k} = Au_y h z_y^\alpha - c - \delta k, \quad (2.1)$$

where c is the amount of good y devoted to consumption, and $\delta \in [0, 1]$ is the depreciation rate of the physical capital stock. The second sector produces consumption good x by means of the production function

$$x = B (s_x k)^\beta (u_x h)^{1-\beta} = B u_x h z_x^\beta, \quad (2.2)$$

where s_x and u_x are the shares of physical and human capital allocated to this sector, respectively, $z_x = s_x k / u_x h$ is the physical to human capital ratio, $B > 0$ is the sectoral *TFP*, and $\beta \in (0, 1)$ measures the intensity of physical capital in this sector. We assume that this sector produces food and services devoted to consumption, such as cultural or entertainment goods. Thus, the output of this sector can only be devoted to consumption. Finally, the third sector produces commodity e by means of the production function

$$e = D [(1 - s_y - s_x) k]^\pi [(1 - u_y - u_x) h]^{1-\pi} = D (1 - u_y - u_x) h z_h^\pi,$$

where $z_h = (1 - s_y - s_x) k / (1 - u_y - u_x) h$ is the physical to human capital ratio, $D > 0$ is the sectoral *TFP*, and $\pi \in (0, 1)$ measures the intensity of physical capital in this sector. This commodity is devoted exclusively to increase the stock of human capital and, therefore, we identify this sector with the education sector. The accumulation of the human capital stock is thus given by

$$\dot{h} = D (1 - u_y - u_x) h z_h^\pi - \eta h, \quad (2.3)$$

where $\eta \in [0, 1]$ is the depreciation rate of human capital.

The economy is populated by an infinitely lived representative agent characterized by the following utility function:

$$U(c, x) = \frac{(c^\theta x^{1-\theta})^{1-\sigma}}{1-\sigma},$$

where the parameter $\theta \in [0, 1]$ measures the share of good c in the composite consumption good $c^\theta x^{1-\theta}$, and $\sigma > 0$ is the (constant) elasticity of the marginal utility of this composite consumption good. We assume that population remains constant and that the representative agent is endowed with k units of physical capital and h units of human capital. Let w be the return from human capital (i.e., the real wage per unit of human capital) and r the return from physical capital (i.e., the real interest rate). We assume perfect sectorial mobility so that the wage and interest rate are independent

of the sector where the representative agent allocates the units of physical and human capital. Accordingly, the consumer budget constraint is given by

$$wh + rk = c + I_k + p_x x + p_h I_h, \quad (2.4)$$

where p_x is the relative price of good x in units of good c , p_h is the relative price of human capital in units of physical capital. Finally, I_h and I_k are the gross investment in human and physical capital, respectively,

$$I_k = \dot{k} + \delta k, \quad (2.5)$$

and

$$I_h = \dot{h} + \eta h. \quad (2.6)$$

The representative agent maximizes

$$\int_0^{\infty} e^{-\rho t} U(c, x) dt, \quad (2.7)$$

subject to (2.4), (2.5), and (2.6), where $\rho > 0$ is the subjective discount rate. The solution to this optimization problem is given by the following equations:¹

$$p_x = \left(\frac{1 - \theta}{\theta} \right) \left(\frac{c}{x} \right), \quad (2.8)$$

$$\frac{\dot{p}_h}{p_h} = r - \frac{w}{p_h} + \eta - \delta, \quad (2.9)$$

$$\frac{\dot{c}}{c} = \frac{r - \rho - \delta}{\sigma} - \left(\frac{(1 - \sigma)(1 - \theta)}{\sigma} \right) \left(\frac{\dot{p}_x}{p_x} \right), \quad (2.10)$$

and the transversality conditions

$$\lim_{t \rightarrow \infty} e^{-\rho t} p_x^{(1-\theta)(\sigma-1)} c^{-\sigma} k = 0, \quad (2.11)$$

and

$$\lim_{t \rightarrow \infty} e^{-\rho t} p_x^{(1-\theta)(\sigma-1)} c^{-\sigma} h = 0. \quad (2.12)$$

Equation (2.8) implies that the price level p_x is equal to the marginal rate of substitution between the two consumption goods. Equation (2.9) shows that the growth of the price p_h is determined by the standard non-arbitrage condition between investments in physical and human capital. Finally, (2.10) characterizes the growth rate of consumption good c . Note that if we define consumption expenditure as $\omega = c + p_x x$ then, using (2.8), we obtain that $\omega = c/\theta$ and, thus, the growth rate of consumption good c coincides with the growth rate of consumption expenditure. Obviously, this result follows because preferences are homothetic.

Equation (2.10) tells us that the growth rate of consumption is not only driven by the diminishing returns to scale but also by the change in the relative price of the two consumption goods. The growth effect of a rise in the interest rate is measure

¹The consumer's maximization problem is solved in the appendix.

by $IES = \frac{1}{\sigma}$ and the growth effect of a rise in the growth rate of prices depends on the elasticity of the marginal utility of the consumption good c with respect to the consumption good x

$$\varepsilon_{c,x} = (1 - \theta)(1 - \sigma).$$

Note that if $\varepsilon_{c,x} = 0$, the growth rate of prices does not affect the growth rate of consumption. This occurs when either there is a unique consumption good, $\theta = 1$, or the two consumption goods are Edgeworth independent, $\sigma = 1$. In contrast, if $\varepsilon_{c,x} < (>) 0$ the two consumption goods are Edgeworth substitutes (complementaries) and a rise in the relative price increases (decreases) consumption expenditure growth. The intuition on this result is as follows. A rise in the relative price p_x reduces the amount of good x consumed. This reduction implies an increase (reduction) in the amount of good c consumed when the two goods are Edgeworth substitutes (complementaries).

Firms maximize profits in each sector and, thus, the competitive factors payment must satisfy simultaneously the following conditions:

$$r = \alpha A z_y^{\alpha-1}, \quad (2.13)$$

$$r = p_x \beta B z_x^{\beta-1}, \quad (2.14)$$

$$r = p_h \pi D z_h^{\pi-1}, \quad (2.15)$$

$$w = (1 - \alpha) A z_y^\alpha, \quad (2.16)$$

$$w = p_x (1 - \beta) B z_x^\beta, \quad (2.17)$$

and

$$w = p_h (1 - \pi) D z_h^\pi. \quad (2.18)$$

Combining the system of equations (2.13) to (2.18), we obtain

$$z_i = \psi_i p_h^{\frac{1}{\alpha-\pi}}, \quad i = y, h, x. \quad (2.19)$$

where

$$\psi_y = \left(\frac{\pi}{\alpha}\right)^{\frac{\pi}{\alpha-\pi}} \left(\frac{1-\alpha}{1-\pi}\right)^{\frac{\pi-1}{\alpha-\pi}} \left(\frac{D}{A}\right)^{\frac{1}{\alpha-\pi}},$$

$$\psi_h = \left(\frac{\pi}{1-\pi}\right) \left(\frac{1-\alpha}{\alpha}\right) \psi_y,$$

and

$$\psi_x = \left(\frac{(1-\alpha)\beta}{\alpha(1-\beta)}\right) \psi_y.$$

Using equation (2.14), (2.15) and (2.19), we obtain

$$p_x = \varphi p_h^{\frac{\alpha-\beta}{\alpha-\pi}},$$

where

$$\varphi = \frac{\pi P (\psi_h)^{\pi-1}}{\beta B (\psi_x)^{\beta-1}}.$$

This relationship between the relative prices implies that

$$\frac{\dot{p}_x}{p_x} = \left(\frac{\alpha - \beta}{\alpha - \pi} \right) \left(\frac{\dot{p}_h}{p_h} \right). \quad (2.20)$$

Equation (2.20) shows that the relationship between the growth of the relative prices only depends on the capital intensity ranking among the different sectors. In particular, the three sector structure of our model nests the following two well-known growth models. On the one hand, when $\alpha = \beta$, the same technology produces the two consumption goods and the transitional dynamics of our model coincides with the transitional dynamics of the two-sector growth model with a unique consumption good, which was analyzed by Uzawa (1965) and Lucas (1988). In this case, the relative price between the two consumption goods remains constant and equal to $p_x = \frac{A}{B}$. This means that the growth of consumption only depends on the interest rate, as in the Uzawa-Lucas model. On the other hand, when $\alpha = \pi$, the same technology produces the two capital stocks and the transition of our model coincides with the transition in models with several consumption goods and a unique capital stock (Rebelo, 1991). In this case, the relative price between the two capital goods remains constant and the growth rate only depends on the interest rate.

We conclude from this analysis that consumption growth depends on the growth of the prices when the two consumption goods are not Edgeworth independent and when the technologies producing the two consumption goods and the two capital stocks are different.

We next characterize the shares of physical and human capital in each sector. To this end, we define the aggregate ratios $z = k/h$ and $q = c/k$. Then, we combine (2.2) and (2.8) to get

$$u_x = \left(\frac{1 - \theta}{\theta} \right) \left(\frac{qz}{p_x B z_x^\beta} \right), \quad (2.21)$$

and we use the definition of z_x to obtain

$$s_x = \left(\frac{1 - \theta}{\theta} \right) \left(\frac{qz_x^{1-\beta}}{p_x B} \right). \quad (2.22)$$

Next, we combine the definitions of z_y and z_h to get

$$u_y = \frac{(1 - u_x) z_h - (1 - s_x) z}{z_h - z_y}, \quad (2.23)$$

and

$$s_y = \left(\frac{z_y}{z} \right) \left(\frac{(1 - u_x) z_h - (1 - s_x) z}{z_h - z_y} \right). \quad (2.24)$$

We proceed to characterize the growth rate of the two capital stocks. For that purpose, we use (2.1) to obtain

$$\frac{\dot{k}}{k} = \frac{A u_y z_y^\alpha}{z} - q - \delta, \quad (2.25)$$

and from (2.3) we get

$$\frac{\dot{h}}{h} = D(1 - u_y - u_x) z_h^\pi - \eta. \quad (2.26)$$

Finally, we obtain the equations that characterize the equilibrium path. First, we combine (2.9), (2.13) and (2.16) to obtain

$$\frac{\dot{p}_h}{p_h} = \underbrace{\alpha A z_y^{\alpha-1} - \frac{(1-\alpha) A z_y^\alpha}{p_h}}_{\kappa(p_h)} + \eta - \delta. \quad (2.27)$$

Note that the right hand side of the previous equation can be written as a function of the relative price p_h , $\kappa(p_h)$, after making use of (2.19).

We combine (2.10) with (2.13) and (2.20) to obtain

$$\frac{\dot{c}}{c} = \underbrace{\frac{\alpha A z_y^{\alpha-1} - \rho - \delta}{\sigma}}_{\nu(p_h)} - \chi \kappa(p_h) \equiv \gamma(p_h), \quad (2.28)$$

where

$$\chi = \left(\frac{(1-\sigma)(1-\theta)}{\sigma} \right) \left(\frac{\alpha - \beta}{\alpha - \pi} \right).$$

Note again that the first term in the right hand side of (2.28) is a function of the relative price p_h , $\nu(p_h)$, as follows from (2.19). Combining (2.25) and (2.26), we get

$$\frac{\dot{z}}{z} = \frac{A u_y z_y^\alpha}{z} - q + \eta - \delta - D(1 - u_y - u_x) z_h^\pi, \quad (2.29)$$

and combining (2.25) and (2.28) we obtain

$$\frac{\dot{q}}{q} = \nu(p_h) - \chi \kappa(p_h) - \frac{A u_y z_y^\alpha}{z} + q + \delta. \quad (2.30)$$

The dynamic equilibrium is thus characterized by a set of paths $\{p_h, z, q\}$ such that, given the initial value of the ratio between the two capital stocks z_0 , solves the equations (2.27), (2.29), and (2.30), and satisfies (2.19), (2.21), (2.22), (2.23) together with the transversality conditions (2.11) and (2.12). As in the standard two-sector growth model, there is a unique state variable, z , and the transition will be governed by the imbalances between the two capital stocks.

Let us combine (2.27), (2.28), (2.13) and (2.16) to obtain that the rate of growth of consumption satisfies

$$\frac{\dot{c}}{c} = \gamma(p_h) = \left(\frac{1}{\sigma} - \chi \right) r + \left(\frac{\chi}{p_h} \right) w - \left(\frac{\rho + \delta}{\sigma} \right) - \chi(\eta - \delta). \quad (2.31)$$

This equation shows that the rate of growth of consumption depends both on the interest rate and on the wage rate when $\chi \neq 0$. In this case, cross-country differences in the growth rates will also be explained by wage differentials.

We define a steady-state equilibrium as an equilibrium path along which the ratios z and q and the relative prices remain constant. The following result characterizes the steady-state equilibrium.

Proposition 2.1. *The unique long-run value p_h^* of the relative price solves $\kappa(p_h^*) = 0$, the two capital stocks and consumption expenditure grow at the same constant growth rate $g^* \equiv \nu(p_h^*)$, and the long run value of the ratio z of capitals and of the ratio q of consumption to capital are unique and equal to*

$$z^* = \frac{\tilde{\varepsilon}(z_x - z_h) + \varepsilon(z_y - z_x)z_h - \tilde{\varepsilon}\left(\frac{z_h - z_y}{Az_y^\alpha}\right)}{\varepsilon(z_x - z_h) + \widehat{\varepsilon}\varepsilon(z_y - z_x) - \frac{z_h - z_y}{Az_y^\alpha}},$$

and

$$q^* = \frac{\widehat{\varepsilon}\tilde{\varepsilon} - z_h}{\tilde{\varepsilon}(z_x - z_h) - \tilde{\varepsilon}\left(\frac{z_h - z_y}{Az_y^\alpha}\right) + \varepsilon(z_y - z_x)z_h},$$

where

$$\varepsilon = \frac{(1 - \theta)}{\theta p_x B z_x^\beta},$$

$$\tilde{\varepsilon} = \left(\frac{g^* + \eta}{P z_h^\pi}\right)(z_h - z_y) + z_y,$$

and

$$\widehat{\varepsilon} = (z_h - z_y) \left(\frac{g^* + \delta}{Az_y^\alpha}\right) + 1.$$

Note that neither the long-run price level p_h^* nor the growth rate g^* depend on the parameter θ measuring relative weight of the consumption goods in the utility function. As in the standard endogenous growth model with a unique consumption good, the long-run values of these two variables only depend on the technology. In contrast, the long run value of the ratios z^* and q^* depend on the parameter θ . On the one hand, as θ increases, the weight of consumption good c in the utility function increases and, as a consequence, the ratio q^* increases in the long run. On the other hand, the change in the patterns of consumption due to an increase in θ also affects the long run value of the ratio of capitals z^* . In particular, when the sector that produces the consumption good c is relatively more (less) intensive in physical than the sector that produces the consumption good x , then the ratio z^* increases (decreases) with θ .

3. Convergence

Let us now analyze how the behavior of the growth rate of consumption during the transition is affected by the introduction of a second consumption good.

Proposition 3.1. *The steady state equilibrium is saddle path stable.*

This result implies that the dynamic equilibrium is unique, which allows us to make comparisons between the growth patterns and to define the concept of asymptotic speed of convergence. Concerning the speed of convergence, in the proof of Proposition 3.1 it is shown that if $\alpha > \pi$ then the speed of convergence is equal to $p_h^* \kappa'(p_h^*)$ and is independent of the parameter θ . In contrast, if $\alpha < \pi$ then the speed of convergence depends on this parameter. In this case, the equilibrium value of the relative price p_h

equals its steady state value and is then constant along the transition. This implies that the consumption growth rate is constant and equal to $v(p_h^*)$ along the transition when $\alpha < \pi$. Therefore, there is no convergence in terms of consumption growth in this case. Following Echevarría (1997), we will assume that $\alpha > \pi$ so that consumption growth will exhibit transitional dynamics.²

We proceed with the analysis of the two different forces governing the transition in this economy. As shown in equation (2.28), these two forces are summarized in the terms $v(p_h)$ and $\kappa(p_h)$, which are functions of the relative price. The function $v(p_h)$ summarizes the growth effects of an increase in the interest rate and $\kappa(p_h)$ is a measure of the growth effects of the relative price. These two functions are decreasing when $\alpha > \pi$. As the two forces only depend on the relative price, the nature of the transition will depend on the slope of the stable manifold relating the price p_h with the state variable z . Equation (2.19) shows that the physical to human capital ratio in the three sectors, z_y , z_x and z_h , depends positively on the relative price p_h when $\alpha > \pi$. An increase in this price changes the sectoral allocation of capital and, moreover, discourages the accumulation of human capital when this activity is relatively more intensive in human capital. Equation (2.19) shows that this accumulation effect always dominates, so that the increase in p_h pushes the relative participation of physical capital in the production of the three sectors up. Therefore, we then conclude that the slope of the stable manifold relating the price p_h with the state variable z is positive. The intuition behind this conclusion is as follows. When $z_0 < (>) z^*$, h_0 is large (small) in comparison to k_0 and then the relative price of human capital will be lower (higher) than its long run value. This implies that the relative price p_h decreases along the transition when $z_0 > z^*$ and increases otherwise. Obviously, this means that the slope of the stable manifold is positive and it also means that $\kappa(p_h) < (>) 0$ when $z_0 > (<) z^*$.

Concerning the characterization of the transition of our economy, we should first mention that the coexistence of two forces determining the transition implies that the dynamic path may exhibit non-monotonicities when these two forces have opposite growth effects. To show these non-monotonicities, we use (2.31), (2.13) and (2.16), to obtain the following derivative of the rate of growth of consumption with respect to the relative price p_h :

$$\frac{\partial \gamma(p_h)}{\partial p_h} = \left(\frac{(1 - \alpha) A z_y^{\alpha-1}}{(\alpha - \pi) p_h} \right) \phi(p_h), \quad (3.1)$$

where

$$\phi(p_h) = \pi \chi \psi_y p_h^{\frac{1-\alpha+\pi}{\alpha-\pi}} - \alpha \left(\frac{1}{\sigma} - \chi \right).$$

Note that if $\chi \in (0, 1/\sigma)$ then there exists a value of p_h such that $\phi(p_h) = 0$. As the price monotonically increases with z , there is a value of z , say \bar{z} , such that $\phi(p_h) > (<) 0$ when $z > (<) \bar{z}$. The following result uses these arguments to provide conditions for the existence of non-monotonic behavior:

Proposition 3.2. *Assume that $\theta \in (0, 1)$. Then,*

²The role of the factor intensity ranking in the transitional dynamics of multi-sector growth models is extensively discussed in Bond et al. (1996).

- (a) If $\chi \leq 0$, the growth rate is monotonically decreasing (increasing) when $z_0 < z^*$ ($z_0 > z^*$).
- (b) If $\chi \in (0, 1/\sigma)$ and $\bar{z} < z^*$, the growth rate monotonically decreases when $z_0 > z^*$, monotonically increases when $z_0 \in (\bar{z}, z^*)$, and it exhibits a non-monotonic behavior when $z_0 < \bar{z}$.
- (c) If $\chi \in (0, 1/\sigma)$ and $\bar{z} \geq z^*$, the growth rate monotonically decreases when $z_0 < z^*$, monotonically increases when $z_0 \in (z^*, \bar{z})$, and it exhibits a non-monotonic behavior when $z_0 > \bar{z}$.
- (d) If $\chi > 1/\sigma$, the growth rate is monotonically increasing (decreasing) when $z_0 < z^*$ ($z_0 > z^*$).

The results in Proposition 3.2 imply that in this economy we can distinguish four types of transition. These different types of transition are represented in Figure 1, where the consumption growth rate is displayed as a function of the ratio of capitals. In particular, Panel 1a shows the consumption growth rate when $\chi = 0$ and consumption growth is not affected by the growth of the relative price p_h . In this case, as in the Uzawa-Lucas model, the consumption growth rate is a monotonic function that decreases when $z_0 < z^*$ and increases when $z_0 > z^*$ (see Mulligan and Sala-i-Martin (1993) for a complete analysis of the transitional dynamics of the Uzawa-Lucas model). In fact, $\chi = 0$ when the production structure of the economy coincides with the one in the Uzawa-Lucas model ($\alpha = \beta$), there is a unique consumption good ($\theta = 1$), or the two consumption goods are Edgeworth independent ($\sigma = 1$). Moreover, the same type of convergence holds when $\chi < 0$. However, when $\chi \in (0, 1/\sigma)$ the patterns of growth are different from the ones in the Uzawa-Lucas model. On the one hand, the consumption growth rate exhibits a non-monotonic behavior when the initial value of the ratio of capitals is sufficiently far from its stationary value. On the other hand, as shown in Panels 1b and 1c, when $\chi > 0$ we must distinguish two types of transition, depending on the relationship between \bar{z} and z^* . Interestingly, when $\bar{z} < z^*$, the local dynamics implies that convergence is from below when $z_0 < z^*$ and from above otherwise. Therefore, in this case, the conclusions from convergence are reversed due to the effect of the growth of prices. As shown in Panel 1d, this reversed transition also arises when $\chi > 1/\sigma$. To see the implications of our analysis, suppose that the economy suffers a reduction in the stock of physical capital that reduces the ratio z of physical to human capital. This reduction implies an initial increase in the growth rate of consumption in a model with a unique consumption good, whereas it implies an initial reduction in the growth rate in our model.

[Insert Figure 1]

The previous analysis and the results in Proposition 3.2 show that the nature of the transition crucially depends on the value of the parameters when there are heterogeneous consumption goods. We next discuss which is the most plausible type of transition. We address this issue through the following simulation. In order to fit our model with data, we will consider that the commodity y is manufacturing, the consumption good x is composed of primary goods and services, and human capital is education. We use the labor income shares in the primary, manufacturing and service

sectors, and the sectoral composition of GDP reported by Echevarria (1997) for the US economy to set $\alpha = 0.34$ and $\beta = 0.49$. We take the average share of physical capital in the final education output estimated by Perli and Sakellaris (1998) and we consider $\pi = 0.18$. We assume $\delta = 0.056$ to replicate that the investment in physical capital amounts to 7.6% of its stock. Moreover, Perli and Sakellaris (1998) pointed out that estimates of the depreciation rate η vary widely. We choose $\eta = 0.025$, which corresponds with the low end of the range. We set arbitrarily $A = B = 1$, and set $D = 0.0851$ to generate a long-run growth rate equals to 0.02. As follows from equation (2.13), the parameter θ measures the fraction of total consumption expenditures devoted to consumption goods produced in the manufacturing sector. According to Kongsamunt et al. (2001), this fraction was roughly constant during the last century and equal to 0.3. We then select this value for the parameter θ . The value of the other two preference parameters, σ and ρ , depends on the value of the *IES*. As the Proposition 3.2 shows, this value crucially determines the nature of the transition, as it provides a measure of the intensity of the first force. We consider three different values of *IES*: 0.5, 0.41 and 0.37. We set the values of σ and ρ that jointly replicates these values for *IES* and an interest rate net of depreciation equal to 5.6%. In the high elasticity economy we obtain $\sigma = 2$ and $\rho = 0.016$, whereas we get $\sigma = 2.4$ and $\rho = 0.008$ for the *IES* = 0.41 economy, and we get $\sigma = 2.7$ and $\rho = 0.002$ for the low elasticity economy. In all these economies we have $\chi > 0$, which implies that the two forces have opposite growth effects. Then, when the second force dominates, the nature of the transition is going to be different from the one in models with a unique consumption good. Figures 2, 3 and 4 show that this occurs when the *IES* is low.

[Insert Figures 2, 3 and 4]

Figure 2 shows the transitional dynamics in the high elasticity economy. The growth rate as a function of the ratio of capitals exhibits an U-shaped curve, which means that the transition is non-monotonic (see Panel 2a). We see that the non-monotonic behavior arises in the economy with heterogenous consumption when the initial value of the ratio of capital is above its long run value. Panels 2b and 2c compare the transition in this economy with the transition in an equivalent economy with a unique consumption good, that is $\theta = 1$. From this comparison, it follows that in both economies convergence is from above when the ratio of capitals is initially smaller than its long run value and it is from below otherwise.

Figure 3 shows the transitional dynamics in the *IES* = 0.41 economy. The transition is also non-monotonic. However, in this case, when the ratio of capitals is initially smaller than its long run value, convergence in our economy with two consumption goods is from below, whereas convergence in the economy with a unique consumption good is from above. When the ratio of capital is initially above its long run value, convergence is from below in our economy with heterogenous consumption and from above in the economy with $\theta = 1$. Therefore, when the *IES* is low, the transition is reversed when heterogeneous consumption goods are introduced. This occurs because the *IES* measures the growth effects of the interest rate. Then, when the *IES* is sufficiently low, the growth effect of changes in the interest rate is low in comparison with the growth effects of changes in the growth of the relative price. In this case, even if the initial values of the economy are close to the corresponding steady-state values,

the transition is different from the one arising in an economy where the transition is governed only by the diminishing returns to capital. This reversed transition is also displayed in Figure 4 that shows the transitional dynamics in the $IES = 0.37$ economy. In this case, the IES is so low that the second force dominates the transition. This implies that the transition is monotonic but reversed when heterogeneous consumption goods are introduced.

[Insert Figures 5, 6 and 7]

In Figures 5, 6 and 7 we analyze the growth effects of a permanent technological shock that reduces the TFP of the manufacturing sector, given by the parameter A , by 50%. These figures display the growth rate of our economy with two consumption goods and the growth rate of an economy with a unique consumption good. Figure 5 compares these growth rates when the $IES = 0.5$. In both economies the patterns of growth implied by these technological shocks are similar. The growth rate initially suffers a strong decline and then it increases until it converges to its new long run value. Obviously, this long run value is smaller than the one before the shock. In the economy with a unique consumption good, the growth rate only depends on the interest rate, which falls due to the technological shock. This reduces investment and then the stock of physical capital declines during the transition. The reduction in the stock of physical capital implies that the interest rate increases during the transition. Note that the behavior of the interest rate explains the initial strong reduction in the growth rate and also the increase in the growth rate during the transition. In the economy with heterogeneous consumption goods, the growth rate also depends on the growth of the relative price between the two capital stocks. As the stock of physical capital decreases during the transition, the relative price of human capital also falls during the transition, which results in an increase in the growth rate as $\chi > 0$. This positive growth effect of prices explains that the reduction in the growth rate shown in Figure 5 is smaller in the economy with heterogeneous consumption goods than in the economy with a unique consumption good. The positive growth effect due to the decline of the relative prices also explains the growth effects of the shock displayed in Figures 6 and 7. In Figure 6 it is assumed that the $IES = 0.41$ and in Figure 7 it is assumed that $IES = 0.37$. In these two cases, in the economy with heterogeneous consumption goods, the growth rate initially increases and then, eventually, decreases until the long run growth rate is attained. This initial positive growth effect is explained by the growth effects of the prices that initially dominate the transition when the IES is sufficiently low.

In Figures 8, 9 and 10 we study the growth effects of the introduction of a 10% income tax rate. We assume that tax revenues are returned to households through a lump-sum subsidy in order to prevent wealth effects. The introduction of this tax reduces the returns from capital, which explains the initial large reduction in the growth rate. Two forces determine the transition towards the new steady state equilibria. On the one hand, the introduction of this tax reduces the accumulation of capital, which causes an increase in the interest rate and in the growth rate during the transition. On the other hand, the service sector increases as agents reduce capital accumulation and then devote more resources to consume. As the service sector is the most capital intensive sector, the demand of capital rises which increases z and reduces the interest

rate and the growth rate during the transition. When $\theta = 1$, this second force is not operative and then the growth rate increases during the transition. However, when $\theta < 1$, the two forces are operative and have opposite growth effects. When the IES is low, the second force dominates the transition and the growth rate decreases as shown in Figure 8. When the IES is high, the two forces compensate and the growth rate is constant along the transition, as shown in Figures 9 and 10.

[Insert Figures 8, 9 and 10]³

We conclude that the growth effects of a shock depend during the transition on the value of θ . This obviously implies that shocks will have different welfare costs depending on the assumptions on preferences. In order to show the welfare cost of a shock, Table 1 provides the uniform increase in consumption necessary to compensate the welfare cost of three different shocks. This table shows how the welfare costs of these shocks change when heterogeneous consumption is introduced. The first shock analyzed is a 10% permanent reduction in the technological parameter A . We show that if $IES = 0.5$ then the uniform increase in consumption should be equal to 36.5% when $\theta = 0.3$ and to 46.1% when $\theta = 1$. Thus, the welfare cost of the shock is 45% larger in the economy with a unique consumption good. Similar results are obtained in the economies with $IES = 0.41$ and $IES = 0.37$. This large differences in terms of welfare are a consequence that the growth effects of this negative shock are smaller in the economy with two consumption goods (see Figures 6, 7, and 8).

[Insert Table 1]

Table 1 also shows the welfare cost of a 10% reduction in the capital stock. As follows from (2.31), when $\theta = 0.3$, this reduction will affect growth through the increase in the interest rate and through the reduction in wages, whereas it will only affect growth through the increase in the interest rate when $\theta = 1$. Thus, as a consequence of this shock, the growth rate is larger in the economy with a unique consumption good (see Figures 3, 4, and 5). This larger growth rate implies that the welfare cost of this shock will be lower in the economy with a unique consumption good. According to Table 1, if $IES = 0.5$ the welfare cost of the reduction in the capital stock is 24.8% when $\theta = 0.3$ and equal to 21.1% when $\theta = 1$. Thus, the welfare cost of the shock is 17% larger in the economy with heterogeneous consumption goods. Similar results are also obtained in the economies with $IES = 0.41$ and $IES = 0.37$.

Finally, Table 1 shows the welfare cost of the introduction of a 10% income tax. The introduction of this tax reduces the growth rate and shifts the composition of consumption towards a higher demand of services. This higher demand of services increases the price of these goods, which causes a welfare cost when $\theta < 0.3$. This price effect is particularly important in the long run and implies a higher welfare cost in those economies with heterogeneous consumption. However, during the transition, the growth rate is smaller in the economies with a unique consumption good. As a consequence, the welfare cost of the shock during the transition is larger when $\theta = 1$. It follows that if the discount rate is sufficiently low, the welfare cost will be larger in economies with homogeneous consumption and, otherwise, the welfare cost is larger when there is a unique consumption good. This explains the results in Table 1 that

show a larger welfare cost in the economies with $\theta = 0.3$ when the *IES* is low and, accordingly, ρ is also low.

4. Concluding remarks

We have analyzed the transitional dynamics of an endogenous growth model with heterogenous consumption goods. We have shown that consumption growth depends both on the interest rate and on the growth of the relative price between the two capitals. Therefore, convergence is determined by two different forces: the diminishing returns to capital and the growth of prices. The existence of these two forces yields interesting differences with respect to the transitional dynamics obtained in the standard growth model with a unique consumption good and two capital stocks. First, we show that, in contrast with the standard growth model, convergence in the growth rate may occur from above if the initial value of the ratio of physical to human capital is larger than its stationary value and may occur from below when the initial value of this ratio is smaller than its stationary value. Second, we show that consumption growth may exhibit a non-monotonic behavior when the two forces have opposite growth effects. These differences in the transition have interesting implications.

On the one hand, economies with the same interest rate may exhibit different growth rates along the transition in our model. Therefore, our model provides an additional explanation to cross-country differences in the growth rates. Rebelo (1992) shows that the introduction of a minimum consumption requirement also implies that the growth rates do not equalize. This occurs because the minimum consumption makes preferences be non-homothetic and then the *IES* is not constant along the transition. In this framework, convergence is driven by the interest rate and by the time-varying *IES*. More recently, Steger (2006) shows that, if there are heterogenous consumption goods and a unique capital stock, then the *IES* is not constant and the growth rates do not equalize. Obviously, he derives this result when preferences are non-homothetic. In contrast, we show that, when there are heterogenous consumption goods, the growth rates are different even with a constant *IES* because of the effect of the growth of the relative capital prices along the transition.

On the other hand, the growth effects of technological shocks depend on the number of consumption goods assumed in the model. This occurs because technological shocks modify both the interest rate and the relative price. Indeed, we show that, if the *IES* is sufficiently low, the initial growth effects of a technological shock can be the opposite from the ones obtained in a model with a unique consumption good. We conclude that the analysis of the business cycle effects of technological shocks must take into account the changes in the relative capital price.

An interesting extension of this paper is to introduce a minimum consumption requirement in one of the consumption goods. In this case, the price of this good will be high in the initial stages of development since the minimum consumption requirement will induce a high marginal utility of this good. Then, as the economy develops, the price will fall sharply until convergence is attained. Therefore, it seems that the introduction of a minimum consumption may accelerate the change of the capital prices and, hence, the introduction of this consumption requirement may increase the effect of the growth of the relative price on consumption growth.

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A. Appendix

Solution to the consumer's optimization problem.

The Hamiltonian function associated to the maximization of (2.7) subject to (2.4), (2.5) and (2.6) is

$$H = e^{-\rho t} \left(\frac{(c^\theta x^{1-\theta})^{1-\sigma}}{1-\sigma} \right) + \lambda (wh + rk - c - I_k - p_x x - p_h I_h) + \mu_1 (I_k - \delta k) + \mu_2 (I_h - \eta h),$$

where λ , μ_1 , and μ_2 are the co-state variables corresponding to the constraints (2.4), (2.5) and (2.6), respectively. The first order conditions are

$$\theta \frac{e^{-\rho t} (c^\theta x^{1-\theta})^{1-\sigma}}{c} - \lambda = 0, \quad (\text{A.1})$$

$$(1-\theta) \frac{e^{-\rho t} (c^\theta x^{1-\theta})^{1-\sigma}}{x} - \lambda p_x = 0, \quad (\text{A.2})$$

$$\lambda = \mu_1, \quad (\text{A.3})$$

$$p_h \lambda = \mu_2, \quad (\text{A.4})$$

$$\lambda r - \delta \mu_1 = -\dot{\mu}_1, \quad (\text{A.5})$$

$$\lambda w - \eta \mu_2 = -\dot{\mu}_2. \quad (\text{A.6})$$

Combining (A.1) and (A.2), we obtain (2.8) and

$$\frac{\dot{x}}{x} = \frac{\dot{c}}{c} - \frac{\dot{p}_x}{p_x}. \quad (\text{A.7})$$

Using (A.3) and (A.4), we obtain that

$$p_h \mu_1 = \mu_2,$$

which implies that

$$\frac{\dot{p}_h}{p_h} + \frac{\dot{\mu}_1}{\mu_1} = \frac{\dot{\mu}_2}{\mu_2},$$

and (2.9) follows from using (A.5) and (A.6). Combining (A.1), (A.3) and (A.5), we obtain that

$$-r + \delta = -\rho + [(1-\sigma)\theta - 1] \left(\frac{\dot{c}}{c} \right) + (1-\sigma)(1-\theta) \left(\frac{\dot{x}}{x} \right),$$

and (2.10) follows from using (A.7). Finally, the transversality conditions (2.11) and (2.12) follow from combining (A.1), (A.3) and (2.8).

Proof of Proposition 2.1. The uniqueness of p_h^* follows from the monotonicity of $\kappa(p_h)$, which can be shown using (2.27)

$$\kappa'(p_h) = - \left(\alpha + \frac{\pi z_y}{p_h} \right) \left(\frac{(1-\alpha) A z_y^{\alpha-1}}{(\alpha-\pi) p_h} \right) > (<) 0 \text{ if } \alpha < (>) \pi.$$

Combining (2.21), (2.22) and (2.23), we obtain

$$u_y = \frac{z_h - z}{z_h - z_y} + \underbrace{\left(\frac{(1-\theta)}{\theta p_x B z_x^\beta} \right)}_{\varepsilon} \left(\frac{z_x - z_h}{z_h - z_y} \right) qz \quad (\text{A.8})$$

and

$$1 - u_y - u_x = \frac{z - z_y}{z_h - z_y} + \varepsilon \left(\frac{z_y - z_x}{z_h - z_y} \right) qz. \quad (\text{A.9})$$

Next, in a steady state, equations (2.26) and (2.25) simplify to

$$\begin{aligned} 1 - u_y - u_x &= \frac{g^* + \eta}{P z_h^\pi}, \\ \frac{A u_y z_y^\alpha}{z} - q &= g^* + \delta. \end{aligned}$$

These two equations can be rewritten as the following system of two equations by using (A.8) and (A.9):

$$\begin{aligned} z + \varepsilon (z_y - z_x) qz &= \underbrace{\left(\frac{g^* + \eta}{P z_h^\pi} \right)}_{\tilde{\varepsilon}} (z_h - z_y) + z_y, \\ z_h + \left(\varepsilon (z_x - z_h) - \frac{z_h - z_y}{A z_y^\alpha} \right) zq &= \underbrace{\left((z_h - z_y) \left(\frac{g^* + \delta}{A z_y^\alpha} \right) + 1 \right)}_{\hat{\varepsilon}} z. \end{aligned}$$

The steady state values of z^* and q^* displayed in Proposition 2.1 are the solutions of this system of equations. ■

Proof of Proposition 3.1. Let J be the Jacobian matrix evaluated at the steady state of the system of differential equations formed by (2.27), (2.29) and (2.30),

$$J = \begin{pmatrix} \frac{\partial \dot{p}_h}{\partial p_h} & \frac{\partial \dot{p}_h}{\partial z} & \frac{\partial \dot{p}_h}{\partial q} \\ \frac{\partial \dot{z}}{\partial p_h} & \frac{\partial \dot{z}}{\partial z} & \frac{\partial \dot{z}}{\partial q} \\ \frac{\partial \dot{q}}{\partial p_h} & \frac{\partial \dot{q}}{\partial z} & \frac{\partial \dot{q}}{\partial q} \end{pmatrix},$$

where

$$\begin{aligned} \frac{\partial \dot{p}_h}{\partial p_h} &= p_h \kappa'(p_h), \\ \frac{\partial \dot{p}_h}{\partial z} &= 0, \\ \frac{\partial \dot{p}_h}{\partial q} &= 0, \end{aligned}$$

$$\begin{aligned} \frac{\partial \dot{z}}{\partial p_h} &= z \left\{ \underbrace{\frac{Az_y^\alpha}{z} \left(\frac{\partial u_y}{\partial p_h} \right) + \frac{Au_y \alpha z_y^{\alpha-1}}{z} \left(\frac{\partial z_y}{\partial p_h} \right)}_{\epsilon_{p_h}} \right. \\ &\quad \left. - Pz_h^\pi \left(\frac{\partial(1-u_y-u_x)}{\partial p_h} \right) - D(1-u_y-u_x) \pi z_h^{\pi-1} \left(\frac{\partial z_h}{\partial p_h} \right) \right\}, \\ \frac{\partial \dot{z}}{\partial z} &= z \left\{ \underbrace{-\frac{Au_y z_y^\alpha}{z^2} + \frac{Az_y^\alpha}{z} \left(\frac{\partial u_y}{\partial z} \right)}_{\epsilon_z} - Pz_h^\pi \left(\frac{\partial(1-u_y-u_x)}{\partial z} \right) \right\}, \\ \frac{\partial \dot{z}}{\partial q} &= z \left\{ \underbrace{\frac{Az_y^\alpha}{z} \left(\frac{\partial u_y}{\partial q} \right)}_{\epsilon_q} - 1 - Pz_h^\pi \left(\frac{\partial(1-u_y-u_x)}{\partial q} \right) \right\}, \\ \frac{\partial \dot{q}}{\partial p_h} &= q \left\{ \frac{\alpha(\alpha-1)Az_y^{\alpha-2}}{\sigma} \left(\frac{\partial z_y}{\partial p_h} \right) - \chi \kappa'(p_h) - \epsilon_{p_h} \right\}, \\ \frac{\partial \dot{q}}{\partial z} &= -q\epsilon_z, \end{aligned}$$

and

$$\frac{\partial \dot{q}}{\partial q} = -q\epsilon_q.$$

The determinant of the Jacobian matrix is

$$Det(J) = \frac{\partial \dot{p}_h}{\partial p_h} \left(\frac{\partial \dot{z}}{\partial z} \frac{\partial \dot{q}}{\partial q} - \frac{\partial \dot{z}}{\partial q} \frac{\partial \dot{q}}{\partial z} \right) = zq\kappa'(p_h) p_h Pz_h^\pi M,$$

where

$$\begin{aligned} M &= \epsilon_q \left(\frac{\partial(1-u_y-u_x)}{\partial z} \right) - \epsilon_z \left(\frac{\partial(1-u_y-u_x)}{\partial q} \right) = \\ &= \left[\frac{\partial u_y}{\partial z} - \frac{Au_y z_y^\alpha}{z^2} \left(\frac{\partial u_y}{\partial q} \right) \right] - \left[\frac{Az_y^\alpha}{z} \left(\frac{\partial u_y}{\partial q} \right) - 1 \right] \frac{\partial u_x}{\partial z} + \left[-\frac{Au_y z_y^\alpha}{z^2} + \frac{Az_y^\alpha}{z} \left(\frac{\partial u_y}{\partial z} \right) \right] \frac{\partial u_x}{\partial q}. \end{aligned}$$

Using (2.23) and (2.24), M simplifies to

$$M = - \left(\frac{1}{z_h - z_y} \right) \left[\underbrace{1 + Az_y^{\alpha-1} \epsilon z_x + \epsilon (z_y - z_x) (Az_y^{\alpha-1} - g^* - \delta)}_+, \right],$$

and the determinant satisfies

$$Det(J) = - \left(\frac{zq\kappa'(p_h) p_h Pz_h^\pi}{z_h - z_y} \right) \left[1 + Az_y^{\alpha-1} \epsilon z_x + \epsilon (z_y - z_x) \left(\frac{g(\sigma - \alpha) + \rho + \delta(1 - \alpha)}{\alpha} \right) \right] < 0,$$

as $\kappa'(p_h) > (<) 0$ when $z_h > (<) z_y$. Next, we obtain the value of the trace

$$\begin{aligned} Tr(J) &= \frac{\partial \dot{p}_h}{\partial p_h} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{q}}{\partial q} = \\ &= p_h \kappa'(p_h) + z \left[\frac{Az_y^\alpha}{z} \left(\frac{\partial u_y}{\partial z} \right) - \frac{Au_y z_y^\alpha}{z^2} - D \frac{\partial(1 - u_y - u_x)}{\partial z} z_h^\pi \right] - q \left[\frac{Az_y^\alpha}{z} \left(\frac{\partial u_y}{\partial q} \right) - 1 \right]. \end{aligned}$$

Using (2.23) and (2.24), the trace simplifies to

$$Tr(J) = \alpha A z_y^{\alpha-1} + \frac{(1-\alpha) A z_y^\alpha}{p_h} - (g + \eta) - (g + \delta).$$

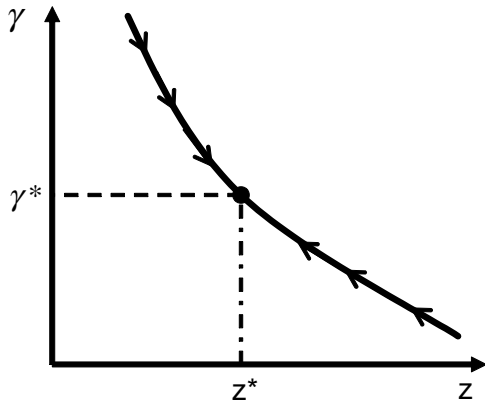
Using $\kappa = 0$, we obtain

$$Tr(J) = 2[(\sigma - 1)g + \rho] > 0,$$

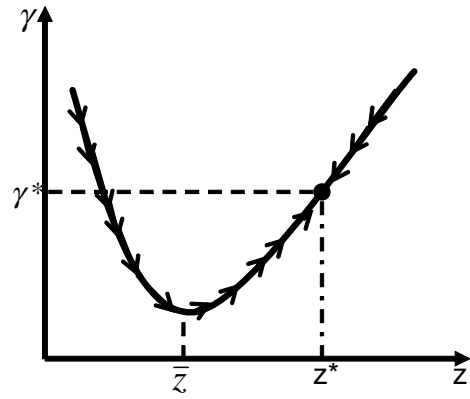
as follows from the transversality condition.

Therefore, the trace is positive, whereas the determinant is negative. This means that there is a unique negative root and that the equilibrium is saddle path stable. Note that the negative root is $p_h \kappa'(p_h)$ when $\alpha > \pi$. Otherwise, the negative roots is one of the roots obtained from the reduced system of differential equations formed by equations (2.29) and (2.30). ■

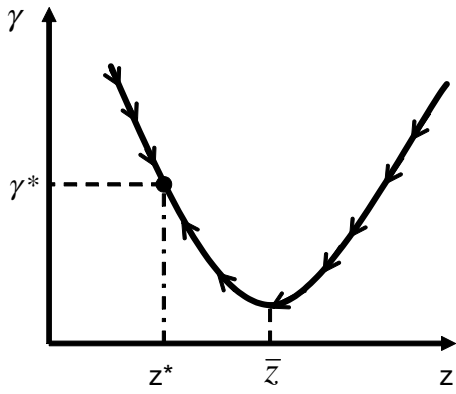
Proof of Proposition 3.2. Parts (a) and (d) follow since the relative price p_h is a monotonic and increasing function of z and $\phi(p_h) > 0$ when $\chi \leq 0$ and $\phi(p_h) < 0$ when $\chi > \frac{1}{\sigma}$. In Part (b), as the slope of the stable manifold is positive, $\phi(p_h) > 0$ along the transition when $z_0 > z^*$ and changes its sign when $z_0 < \bar{z} < z^*$. In the first case, the consumption growth rate is monotonically decreasing, whereas it exhibits a non-monotonic behavior when $z_0 < \bar{z}$. In particular, if $z_0 < \bar{z}$ the growth rate initially decreases and then, as the economy converges to the steady state, it increases. In Part (c), $\phi(p_h) > 0$ along the transition when $z_0 < z^*$ and changes its sign when $z_0 > \bar{z} \geq z^*$. In the first case, the consumption growth rate is monotonically decreasing, whereas it exhibits a non-monotonic behavior when $z_0 > \bar{z}$. In particular, if $z_0 > \bar{z}$ the growth rate initially decreases and, as the economy converges to the steady state, it increases. ■



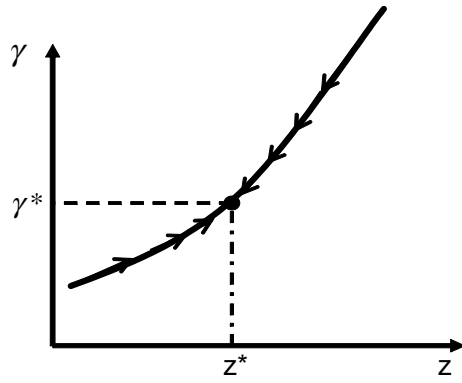
Panel 1a. $\chi \leq 0$.



Panel 1b. $\chi > 0$ and $\bar{z} < z^*$.

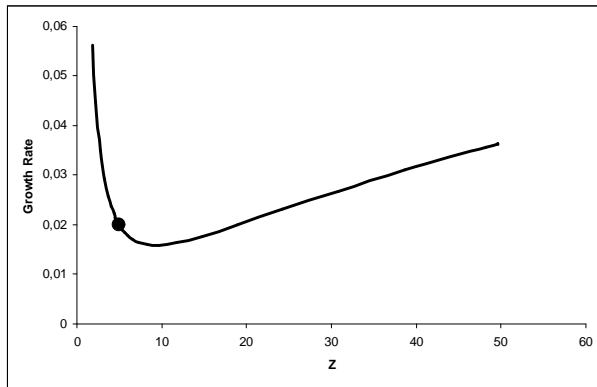


Panel 1c. $\chi > 0$ and $\bar{z} > z^*$.

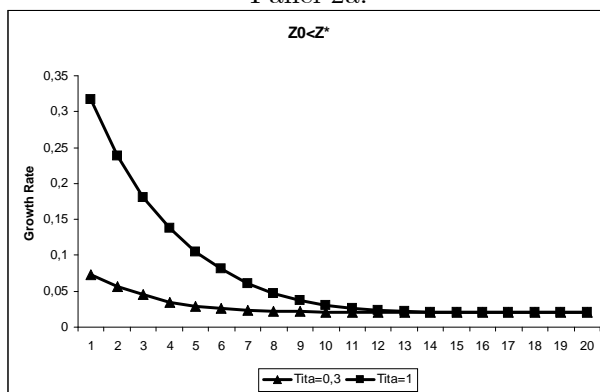


Panel 1d. $\chi > \frac{1}{\sigma}$.

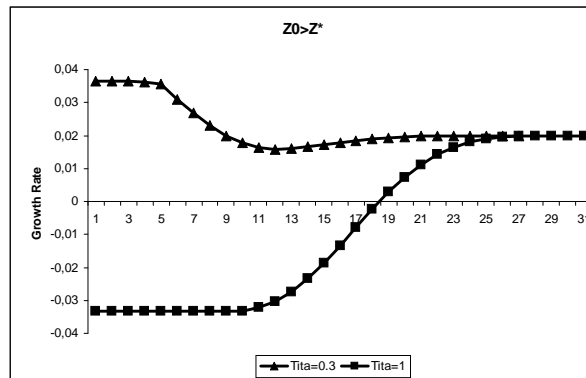
Figure 1. Transition of the consumption growth rate



Panel 2a.

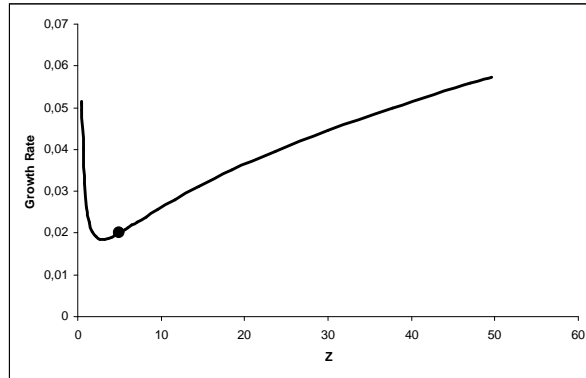


Panel 2b.

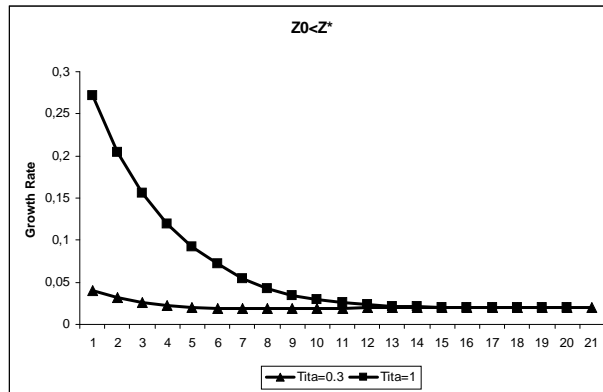


Panel 2c.

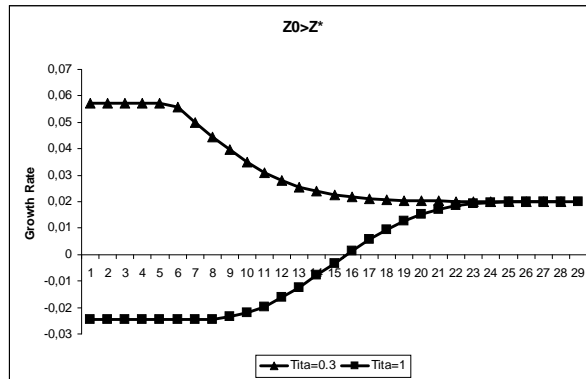
Figure 2. $IES = 0.5$



Panel 3a.

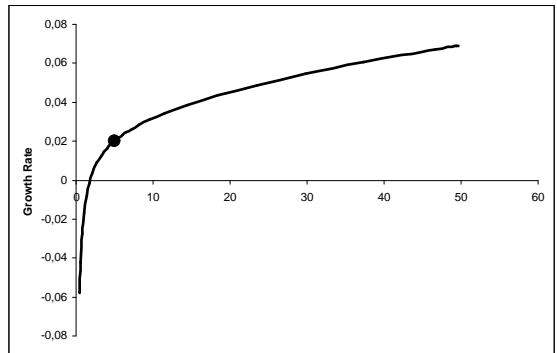


Panel 3b.

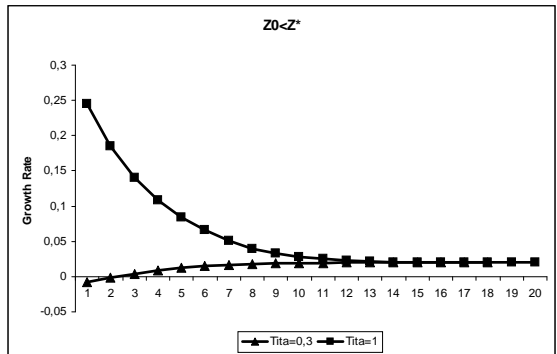


Panel 3c.

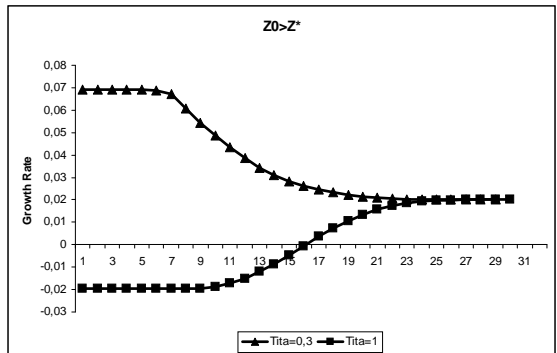
Figure 3. $IES = 0.41$



Panel 4a.



Panel 4b.



Panel 4c.

Figure 4. $IES = 0.37$

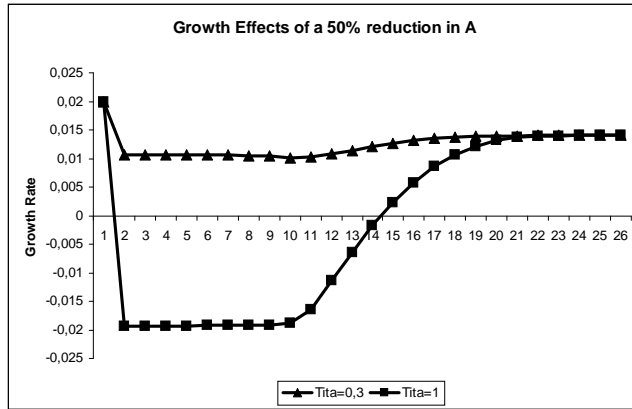


Figure 5. $IES = 0.5$

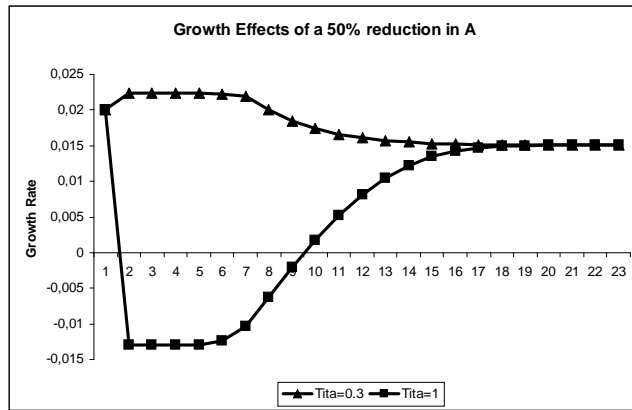


Figure 6. $IES 0.41$

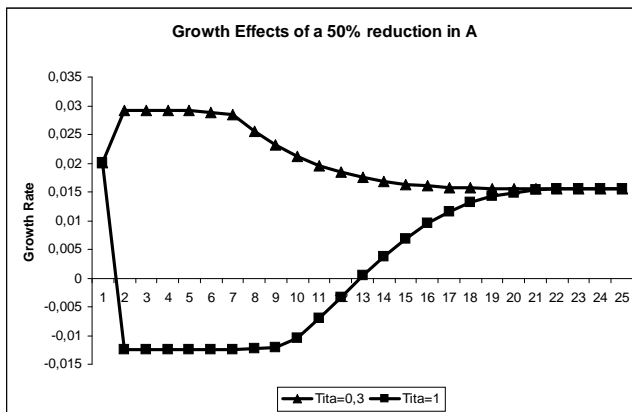


Figure 7. $IES 0.37$

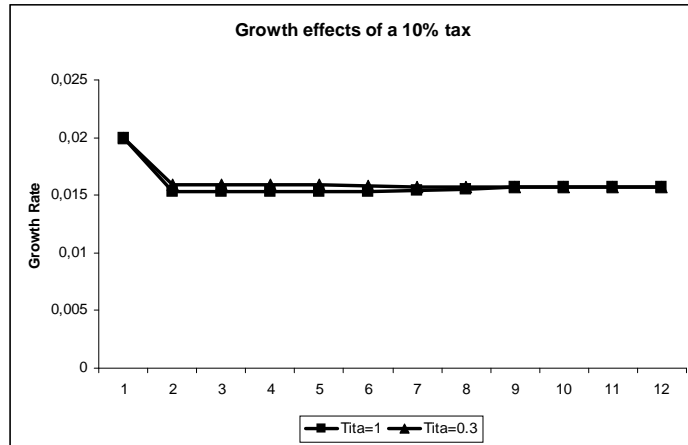


Figure 8. $IES = 0.5$

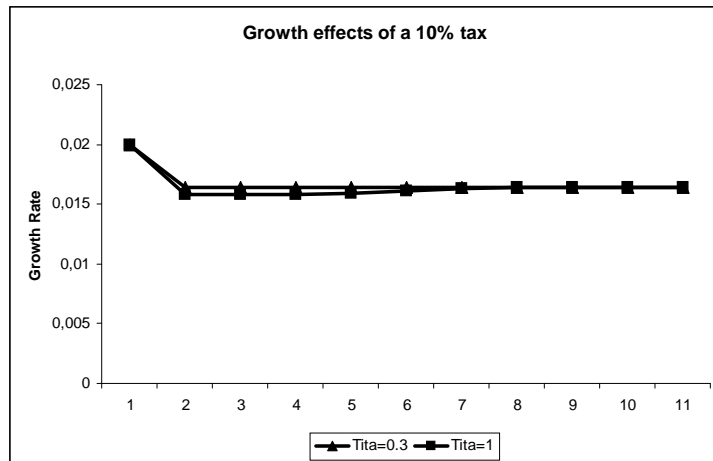


Figure 9. $IES = 0.41$

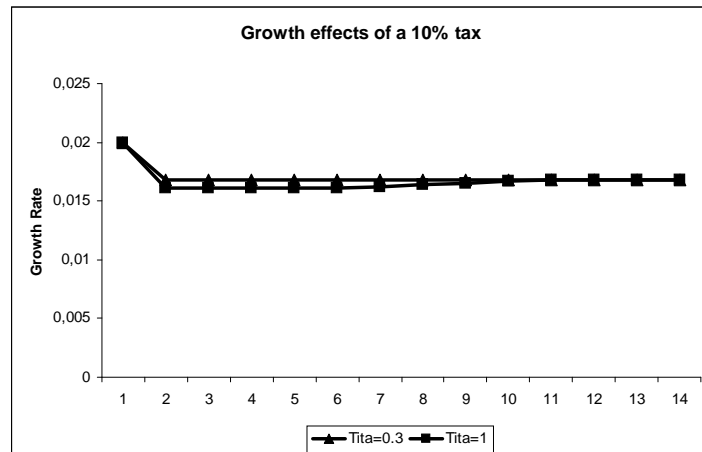


Figure 10. $IES = 0.37$

Table 1. Welfare cost

	<u>$IES = 0.5$</u>		<u>$IES = 0.41$</u>		<u>$IES = 0.37$</u>	
	<u>$\theta = 0.3$</u>	<u>$\theta = 1$</u>	<u>$\theta = 0.3$</u>	<u>$\theta = 1$</u>	<u>$\theta = 0.3$</u>	<u>$\theta = 1$</u>
$\Delta A = -10\%$	36.5%	46.1%	34.2%	43.5%	35.7%	48.9%
$\Delta k = -10\%$	24.8%	21.1%	25.9%	23.2%	27.9%	19.7%
$\Delta \tau = 10\%$	30.3%	50.1%	30.4%	30.7%	40.7%	25.9%